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Scattering of light in cholesteric liquid crystals with large pitch

E. V. Aksenova,* V. P. Romanov,† and A. Yu. Val'kov‡
Department of Physics, Saint Petersburg State University, Ul'yanovskaya, 1, Petrodvoretz, 198504 St. Petersburg, Russia
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We consider the problem of light scattering in a slab of cholesteric liquid crystal with the pitch which significantly exceeds the wavelength of light. The electromagnetic wave propagation and the Green's function are investigated for this medium basing on geometric optics approximation. The correlation function of the director fluctuations is calculated with the aid of the vector analog of the WKB approximation. A general approach to treatment of single light scattering in a stratified medium with smoothly varying properties based on the Kirchhoff method is developed. Angular and polarization dependencies of the single light scattering intensity as well as extinction of the mean field are analyzed. Unusual dependence of the light scattering intensity on the size of the system is found.

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I. INTRODUCTION

Investigation of optical properties and light scattering in helical structures are particularly interesting both for theory and in applications. Recently these problems became important in connection with extensive use of twisted liquid crystals (LCs) in information mapping and especially in LC displays.

In order to solve the problem of light scattering one has to describe the incident field (normal waves in the medium), to calculate the Green's function of the electromagnetic field and to obtain the correlation function of the permittivity fluctuations. In media with spatial inhomogeneities each of these problems presents a serious mathematical obstacle. In the present work we consider these problems for the cell of the cholesteric liquid crystal (CLC) with the pitch which significantly exceeds the wavelength. On the basis of the obtained solutions we consider the problem of light scattering by the cell of CLC.

Cholesteric liquid crystals are systems with one-dimensional periodicity. The regular spatial structure leads to unusual optical properties such as selective reflection and anomalously large optical activity. The problem of electromagnetic wave propagation in the media with one-dimensional periodicity has been a subject of a huge number of studies. The problem leads to a set of differential equations with periodical coefficients which has the exact solution for special cases only. In CLC the only particular case is the wave propagation along the spiral axis [1–3]. The formal analytical solution for an oblique incidence [4–6] has a form of the infinite series and it appears to be difficult for the analysis. Therefore various approximate methods are widely used in optics of layered liquid crystals [7,8]. The emphasis was concentrated on cases with the wavelength being close to the period of the structure and in this case methods developed in x-ray diffraction theory are effective [8]. So far such an approach was the only one used in CLC studies [9–12]. In this case the existence of forbidden zones is typical. The normal waves, the Green's function of the electromagnetic field [13–16] and the spatial correlation function of the director fluctuations [17–20] were studied also.

In the opposite case when the wavelength is much less than the characteristic size of LC structure no appreciable attention was paid so far. However, this case becomes important due to application of nematic twist cells and CLC with the large pitch in information mapping.

It is well known that when an electromagnetic wave propagates along the spiral axis the Mauguin's adiabatic regime takes place, i.e., the polarization of the wave rotates together with optical axis [21]. In the general case of oblique incidence it is relevant to use the WKB (Wentzel-Kramers-Brillouin) method as long as the size of inhomogeneities is much greater than the wavelength. Direct application of the WKB method for electromagnetic waves is difficult since it leads to a system of several coupled equations [4,5]. For CLC with the large pitch the generalization of the WKB method was suggested in Ref. [22]. It allowed one to obtain the analytical solution for oblique incidence of light and, in particular, to get the normal waves of the problem. On the basis of this method the Green's function in such a medium has been obtained in Refs. [23–25] as well.

In CLC the director fluctuations yield the main contribution to scattering. The problem of the director thermal fluctuations in CLC was considered for fluctuations with characteristic scale greater than the pitch ("smecticlike" CLC) [17–20]. The opposite case of short wavelength fluctuations ("nematiclike" CLC) has been recently studied in Ref. [26] using the vector generalization of the WKB method.

In this work we developed a general scheme of calculation of light scattering intensity in CLC with the pitch significantly exceeding the wavelength. The approach based on the Kirchhoff method provides explicit expressions for angular and polarization dependencies of single light scattering intensity and the extinction coefficient. The obtained results are presented in the form convenient for comparison with the experiment.

*Electronic address: aksev@mail.ru
†Electronic address: V.Romanov@pobox.spbu.ru
‡Also at Saint Petersburg Institute for Foreign Economic Relations, Economics and Law, Liteiny 42, Saint Petersburg 191104, Russia. Electronic address: alexvalk@mail.ru

The paper is organized as follows. In Sec. II we present the basic equations describing the elastic energy, fluctuations, and the propagation of electromagnetic waves in CLC. In Sec. III the normal waves of CLC with the large scale periodicity are considered. Maxwell equations are solved in the framework of the geometric optics approximation. Section IV concerns the construction of the Green's function of the electromagnetic field using the normal waves of the medium. In Sec. V our general approach based on the Kirchhoff method for calculation of the light scattering by a slab of CLC is presented. In Sec. VI we calculate the light scattering intensity and analyze various geometries of the experiment. Section VII is devoted to the calculation of the extinction coefficient due to the light scattering. In the conclusion (Sec. VIII) we discuss the obtained results. The spatial correlation function of the director fluctuations in CLC is presented in Appendix A. In Appendix B the surface corrections for the light scattering intensity in a homogeneous medium are calculated. The obtained formulas are compared with the Kirchhoff method results.

II. BASIC EQUATIONS

The elastic free energy of the cholesteric liquid crystal has the form [27]

$$F = \frac{1}{2}\int d\mathbf{r}[K_{11}(\mathrm{div}\,\mathbf{n})^2 + K_{22}(\mathbf{n}\cdot\mathrm{curl}\,\mathbf{n} + q_0)^2 + K_{33}(\mathbf{n}\times\mathrm{curl}\,\mathbf{n})^2], \tag{2.1}$$

where K_{ll} $(l=1,2,3)$ are the Frank modules. The unit vector director $\mathbf{n}=\mathbf{n}(\mathbf{r})$ describes the local orientation of the long axes of molecules. In the equilibrium the energy (2.1) is minimal for helical distribution of the director,

$$\mathbf{n}^0(\mathbf{r}) \equiv \mathbf{n}^0(z) = (\cos\phi, \sin\phi, 0). \tag{2.2}$$

Here we introduced the Cartesian coordinate system with the z axis directed along the CLC axis, $\phi=\phi(z)=q_0 z+\phi_0$, the angle ϕ_0 determines the orientation of the director in the plane $z=0$, $q_0=\pi/P$, P is the pitch. The director $\mathbf{n}^0(\mathbf{r})$ in Eq. (2.2) is normal to the z axis and rotates uniformly around it.

The permittivity tensor $\hat{\varepsilon}$ describes the optical properties of cholesterics. For CLC in equilibrium it has the form [27]

$$\varepsilon^0_{\alpha\beta}(\mathbf{r}) \equiv \varepsilon^0_{\alpha\beta}(z) = \varepsilon_\perp \delta_{\alpha\beta} + \varepsilon_a n^0_\alpha(z) n^0_\beta(z), \tag{2.3}$$

where $\varepsilon_a=\varepsilon_\parallel-\varepsilon_\perp$, $\varepsilon_\parallel,\varepsilon_\perp$ are the permittivities along and perpendicular to $\mathbf{n}^0$, respectively. In the general case $\hat{\varepsilon}(\mathbf{r}) =\hat{\varepsilon}^0(\mathbf{r})+\delta\hat{\varepsilon}(\mathbf{r})$ where $\delta\hat{\varepsilon}(\mathbf{r})$ is the fluctuation of the permittivity tensor.

The wave equation in a nonmagnetic medium for a monochromatic wave is

$$[\mathrm{curl}\,\mathrm{curl} - k_0^2\hat{\varepsilon}(\mathbf{r})]\mathbf{E}(\mathbf{r}) = 0, \tag{2.4}$$

where $\mathbf{E}$ is the electric field, $k_0=\omega/c$, ω is the circular frequency, c is the light velocity in vacuum. The wave equation (2.4) in the integral form is written as

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^0(\mathbf{r}) + k_0^2\int \hat{T}^0(\mathbf{r},\mathbf{r}')\delta\hat{\varepsilon}(\mathbf{r}')\mathbf{E}(\mathbf{r}')d\mathbf{r}'. \tag{2.5}$$

The electric field $\mathbf{E}^0(\mathbf{r})$ and the Green's function of electromagnetic field $\hat{T}^0(\mathbf{r},\mathbf{r}')$ obey the equations

$$[\mathrm{curl}\,\mathrm{curl} - k_0^2\hat{\varepsilon}^0(z)]\mathbf{E}^0(\mathbf{r}) = 0, \tag{2.6}$$

$$[\mathrm{curl}\,\mathrm{curl} - k_0^2\hat{\varepsilon}^0(z)]\hat{T}^0(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}')\hat{I}. \tag{2.7}$$

Here $\hat{I}$ is the unit matrix.

Since Eq. (2.6) is homogeneous the field $\mathbf{E}^0(\mathbf{r})$ can be written as a linear combination of the normal waves. In order to make the problem unambiguous Eq. (2.7) should be supplemented by the corresponding boundary conditions. In the infinite medium they are radiation conditions [28]. Due to symmetry of CLC with respect to displacements in the xy plane we have $\hat{T}^0(\mathbf{r},\mathbf{r}') \equiv \hat{T}^0(\mathbf{r}_\perp-\mathbf{r}'_\perp; z,z')$, where $\mathbf{r}_\perp=(x,y)$.

The second term in the right hand side of Eq. (2.5) corresponds to the scattered field $\mathbf{E}^{(s)}$, produced by the incident field $\mathbf{E}^0$. Solving this equation by iterations and restricting ourselves to the lowest order in $\delta\hat{\varepsilon}$ we obtain the scattered field $\mathbf{E}^{(s)}$ in the Born (single-scattering) approximation

$$\mathbf{E}^{(s)}(\mathbf{r}) = k_0^2\int \hat{T}^0(\mathbf{r}_\perp-\mathbf{r}'_\perp; z,z')\delta\hat{\varepsilon}(\mathbf{r}')\mathbf{E}^0(\mathbf{r}')d\mathbf{r}'. \tag{2.8}$$

The properties of the scattered light are determined by the function of coherence,

$$\langle E^{(s)}_\alpha(\mathbf{r}_1)E^{(s)*}_\beta(\mathbf{r}_2)\rangle = k_0^4\int T^0_{\alpha\gamma}(\mathbf{r}_{1\perp}-\mathbf{r}'_{1\perp}; z_1,z'_1) \times T^{0*}_{\beta\zeta}(\mathbf{r}_{2\perp}-\mathbf{r}'_{2\perp}; z_2,z'_2)\mathcal{G}_{\gamma\nu\zeta\mu}(\mathbf{r}'_1,\mathbf{r}'_2) \times E^0_\nu(\mathbf{r}'_1)E^{0*}_\mu(\mathbf{r}'_2)d\mathbf{r}'_1 d\mathbf{r}'_2, \tag{2.9}$$

where

$$\mathcal{G}_{\gamma\nu\zeta\mu}(\mathbf{r}'_1,\mathbf{r}'_2) = \langle \delta\varepsilon_{\gamma\nu}(\mathbf{r}'_1)\delta\varepsilon^*_{\zeta\mu}(\mathbf{r}'_2)\rangle$$

is the permittivity correlation function, the brackets $\langle\cdots\rangle$ and the star designate the statistical averaging and complex conjugation respectively. Due to CLC symmetry we have

$$\hat{\mathcal{G}}(\mathbf{r}'_1,\mathbf{r}'_2) \equiv \hat{\mathcal{G}}(\mathbf{r}'_{1\perp}-\mathbf{r}'_{2\perp}; z'_1,z'_2).$$

In liquid crystals the director fluctuations

$$\delta\mathbf{n}(\mathbf{r}) = \mathbf{n}(\mathbf{r}) - \mathbf{n}^0(z) \tag{2.10}$$

are most essential in $\delta\hat{\varepsilon}$ [27] and in this paper we consider these fluctuations only. In the framework of this approximation both the equilibrium and the fluctuating permittivity tensor has the form similar to Eq. (2.3) with substitution $\mathbf{n}^0(z) \to \mathbf{n}(\mathbf{r})$:

$$\varepsilon_{\alpha\beta}(\mathbf{r}) \equiv \varepsilon_\perp\delta_{\alpha\beta} + \varepsilon_a n_\alpha(\mathbf{r})n_\beta(\mathbf{r}). \tag{2.11}$$

In the first order the permittivity fluctuations in CLC have the form

$$\delta\varepsilon_{\alpha\beta}(\mathbf{r}) = \varepsilon_a[n_\alpha^0(z)\,\delta n_\beta(\mathbf{r}) + \delta n_\alpha(\mathbf{r})n_\beta^0(z)]. \qquad (2.12)$$

The relationship between corresponding correlation functions is

$$\mathcal{G}_{\alpha\beta\gamma\delta}(\mathbf{r}_\perp;z,z') = \varepsilon_a^2[n_\alpha^0(z)n_\gamma^0(z')g_{\beta\delta}(\mathbf{r}_\perp;z,z') + n_\alpha^0(z)n_\delta^0(z')g_{\beta\gamma}(\mathbf{r}_\perp;z,z') + n_\beta^0(z)n_\gamma^0(z')g_{\alpha\delta}(\mathbf{r}_\perp;z,z') + n_\beta^0(z)n_\delta^0(z')g_{\alpha\gamma}(\mathbf{r}_\perp;z,z')]. \qquad (2.13)$$

Here

$$g_{\alpha\beta}(\mathbf{r}_{1\perp}-\mathbf{r}_{2\perp};z_1,z_2) = \langle\delta n_\alpha(\mathbf{r}_{1\perp},z_1)\,\delta n_\beta(\mathbf{r}_{2\perp},z_2)\rangle \qquad (2.14)$$

is the correlation function of the director fluctuations. Thus for calculation of the single light scattering intensity we should find the normal waves $\mathbf{E}^0$, the Green's function $\hat{T}^0$, and the permittivity correlation function $\hat{\mathcal{G}}$. In the following two sections we are concerned with the first and the second problems.

III. NORMAL WAVES IN CLC WITH THE LARGE PITCH

Let us consider the problem of electromagnetic wave propagation in CLC with the large scale periodicity, $\lambda \ll P$, where λ is the wavelength of light, so we have a large parameter $\Omega = k_0/q_0 = 2P/\lambda \gg 1$. In this case it is reasonable to suppose that the electric field has the form of a quasiplane wave,

$$\mathbf{E}(\mathbf{r}) = \mathbf{A}(\mathbf{r})\exp[i\Psi(\mathbf{r})], \qquad (3.1)$$

where $\Psi(\mathbf{r})$ is the real phase, $\mathbf{A}(\mathbf{r}) = A(\mathbf{r})\mathbf{e}(\mathbf{r})$, $\mathbf{e}(\mathbf{r})$ is the unit vector of polarization, $\mathbf{e}\cdot\mathbf{e}^* = 1$, and $A(\mathbf{r})$ is the real amplitude. At a distance of the order of λ variations of functions $A(\mathbf{r})$, $\mathbf{e}(\mathbf{r})$ and $\Psi'(\mathbf{r})$ are small compared to the functions themselves. In this section we consider CLC in equilibrium only, so the upper index "0" in the field components will be omitted. Substituting Eq. (3.1) into the wave equation (2.6) we get

$$\nabla\times\nabla\times\mathbf{A}(\mathbf{r}) + i[\mathbf{k}(\mathbf{r})\times\nabla\times\mathbf{A}(\mathbf{r}) + \nabla\times\mathbf{k}(\mathbf{r})\times\mathbf{A}(\mathbf{r})] - \mathbf{k}(\mathbf{r})\times\mathbf{k}(\mathbf{r})\times\mathbf{A}(\mathbf{r}) - k_0^2\hat{\varepsilon}^0(\mathbf{r})\mathbf{A}(\mathbf{r}) = 0, \qquad (3.2)$$

where the three-dimensional wave vector $\mathbf{k}(\mathbf{r}) = \nabla\Psi(\mathbf{r})$ is introduced.

In comparison to the first term the second and the third terms are of the order of Ω, the fourth and the fifth terms are of the order of Ω^2 in Eq. (3.2). This hierarchy makes it possible to use the geometric optics approximation. If we keep the principal terms only ($\sim\Omega^2$) then we get the vector analog of the eikonal equation. In this approximation we can calculate $\nabla\Psi = \mathbf{k}$ and the polarization vector $\mathbf{e}$. The terms of the order of Ω yield the so-called transfer equation. This equation makes it possible to determine the wave amplitude $A(\mathbf{r})$.

According to Eq. (3.2) the eikonal equation has the form

$$\mathbf{k}(\mathbf{r})\times\mathbf{k}(\mathbf{r})\times\mathbf{e}(\mathbf{r}) + k_0^2\hat{\varepsilon}^0(\mathbf{r})\mathbf{e}(\mathbf{r}) = 0. \qquad (3.3)$$

For each fixed point $\mathbf{r}$ Eq. (3.3) coincides formally with the ordinary equation describing the propagation of plane waves in homogeneous anisotropic media [29], so that it is possible to use well known results. Particularly in order to have a nonzero solution of Eq. (3.3) the determinant of the matrix in the left hand side has to be equal to zero.

From Eq. (2.4) the conservation law follows:

$$\mathrm{div}\,\mathbf{S} = 0, \qquad (3.4)$$

where

$$\mathbf{S}(\mathbf{r}) = \frac{c}{8\pi k_0}[\mathbf{k}|E|^2 - \mathbf{E}^*(\mathbf{E}\cdot\mathbf{k})] \qquad (3.5)$$

is the Poynting vector [29]. Note that in our case $\hat{\varepsilon}^0(\mathbf{r}) = \hat{\varepsilon}^0(z)$ and the medium is homogeneous in the (x,y) plane. Then $\mathbf{k}(\mathbf{r}) = \mathbf{k}(z) = (\mathbf{q}, k_z(z))$.

Equation (3.3) for uniaxial homogeneous media for the fixed $\mathbf{k}$ direction has two well known solutions corresponding to ordinary and extraordinary waves. In particular the module of the wave vector $\mathbf{k}$ has the form

$$k^{(1)} = k_0\sqrt{\varepsilon_\perp} = k_0 n_{(1)},$$

$$k^{(2)}(z) = k_0\sqrt{\frac{\varepsilon_\perp\varepsilon_\parallel}{\varepsilon_\perp + \varepsilon_a\cos^2\theta}} = k_0 n_{(2)}(z), \qquad (3.6)$$

where θ is the angle between $\mathbf{n}^0(z)$ and $\mathbf{k}(z)$, $n_{(1)}$ and $n_{(2)}(z)$ are refractive indices of the ordinary and extraordinary waves, respectively. In Eq. (3.3) polarization vectors $\mathbf{e}^{(1)}(z)$ and $\mathbf{e}^{(2)}(z)$ corresponding to these values of the $\mathbf{k}$ vectors are determined by conditions

$$\mathbf{e}^{(1)}(z)\perp\mathbf{n}^0(z), \quad \mathbf{e}^{(1)}(z)\perp\mathbf{k}^{(1)}(z),$$

$$\hat{\varepsilon}^0(z)\mathbf{e}^{(2)}(z)\perp\mathbf{k}^{(2)}(z), \qquad (3.7)$$

the vector $\mathbf{e}^{(2)}(z)$ is in the plane formed by vectors $\mathbf{k}^{(2)}(z)$ and $\mathbf{n}^0(z)$, i.e.,

$$\mathbf{e}^{(1)}(z)\parallel\mathbf{k}\times\mathbf{n}^0$$

$$\mathbf{e}^{(2)}(z)\parallel\mathbf{n}^0(\mathbf{k}\hat{\varepsilon}^0\mathbf{k}) - \mathbf{k}(\mathbf{k}\hat{\varepsilon}^0\mathbf{n}^0). \qquad (3.8)$$

Let us present the $\mathbf{k}^{(j)}(z)$ vector in the form $\mathbf{k}^{(j)}(\mathbf{q},z) = (\mathbf{q}, \pm k_z^{(j)}(\mathbf{q},z))$. Taking into account that $\cos\theta = \mathbf{k}^{(2)}(z)\cdot\mathbf{n}^0(z)/k^{(2)}(z) = \mathbf{q}\cdot\mathbf{n}^0(z)/k^{(2)}(\mathbf{q},z)$ one can see that the second expression of Eq. (3.6) for given $\mathbf{q}$ becomes an algebraic equation with respect to $k_z^{(2)}(\mathbf{q},z)$. Thus we have

$$k_z^{(1)}(\mathbf{q},z) \equiv k_z^{(1)}(q) = \sqrt{\varepsilon_\perp k_0^2 - q^2},$$

$$k_z^{(2)}(\mathbf{q},z) = \sqrt{\varepsilon_\parallel k_0^2 - q^2 - \frac{\varepsilon_a}{\varepsilon_\perp}[\mathbf{q}\cdot\mathbf{n}^0(z)]^2}. \qquad (3.9)$$

The signs "+" and "−" correspond to waves propagating in the positive and the negative z direction, respectively. Note that according to Eq. (3.8) polarization vectors $\mathbf{e}^{(j)}$ depend on

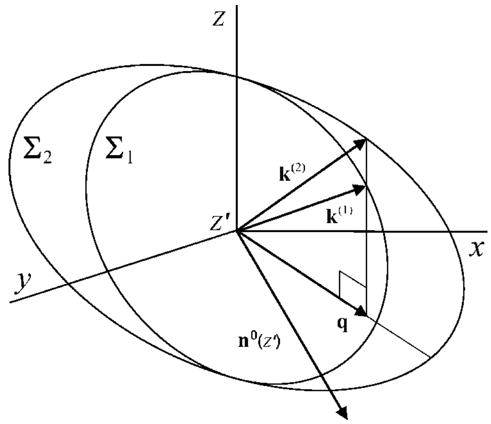


FIG. 1. Two wave vectors $\mathbf{k}^{(1)}$ and $\mathbf{k}^{(2)}$ corresponding to given $\mathbf{q}$. Here circle Σ_1 and ellipse Σ_2 are the cross sections of the surfaces of ordinary (1) and extraordinary (2) wave vectors by the plane containing $\mathbf{q}$ and z axis.

$\mathbf{q}$ also. Figure 1 shows schematically two solutions (3.6) for a fixed $\mathbf{q}$ vector.

Thus we get four normal waves in the form

$$\mathbf{E}_{\pm}^{(j)}(\mathbf{r}) = A^{(j)}(\mathbf{q};z,z_0)\mathbf{e}^{(j)}(\mathbf{q},z) \times \exp\left(i\mathbf{q}\cdot\mathbf{r}_{\perp} \pm i\int_{z_0}^{z} k_z^{(j)}(\mathbf{q},z')dz' \right) \quad (3.10)$$

with the undetermined amplitudes $A^{(j)}(\mathbf{q};z,z_0)$.

Contrary to the standard relations for the homogeneous medium values of $\mathbf{k}^{(2)},\mathbf{e}^{(1)}$, and $\mathbf{e}^{(2)}$ entering Eqs. (3.6)–(3.10) depend on z. Note that polarization vectors in Eq. (3.8) are real in this approximation.

For waves (3.10) we get

$$\mathbf{S}^{(j)}(\mathbf{q},z) = \frac{c}{8\pi k_0}A^{(j)2}(\mathbf{q};z,z_0)\{\mathbf{k}^{(j)}(\mathbf{q},z) - \mathbf{e}^{(j)}(\mathbf{q},z) \times[\mathbf{e}^{(j)}(\mathbf{q},z)\cdot\mathbf{k}^{(j)}(\mathbf{q},z)]\}. \quad (3.11)$$

In our case we have the conservation law in the form $\operatorname{div}\mathbf{S}^{(j)} = \partial_z S_z^{(j)}(\mathbf{q},z) = 0$. Therefore the component $S_z^{(j)}(\mathbf{q},z)$ does not depend on z. Then the amplitudes $A^{(j)}(\mathbf{q};z,z_0)$ can be written in the form

$$A^{(j)}(\mathbf{q};z,z_0) = E_0^{(j)}\frac{B^{(j)}(\mathbf{q},z)}{B^{(j)}(\mathbf{q},z_0)}, \quad (3.12)$$

where

$$B^{(j)}(\mathbf{q},z) = \sqrt{\frac{k_0}{k_z^{(j)}(\mathbf{q},z)}}\frac{\sqrt{\varepsilon_j}}{n_{(j)}(\mathbf{q},z)\cos\delta_{(j)}(\mathbf{q},z)}, \quad (3.13)$$

$\varepsilon_1 = \varepsilon_{\perp}$, $\varepsilon_2 = \varepsilon_{\parallel}$, $\delta_{(j)}(\mathbf{q},z)$ is the angle between the $\mathbf{E}^{(j)}$ and $\mathbf{D}^{(j)} = \hat{\varepsilon}\mathbf{E}^{(j)}$ vectors. For the ordinary beam

$$\cos\delta_{(1)} = 1,$$

for the extraordinary beam

$$\cos\delta_{(2)} = \frac{(\mathbf{e}^{(2)}\hat{\varepsilon}^0\mathbf{e}^{(2)})^{1/2}}{n_{(2)}} = \frac{\varepsilon_{\perp}\sin^2\theta + \varepsilon_{\parallel}\cos^2\theta}{\sqrt{\varepsilon_{\perp}^2\sin^2\theta + \varepsilon_{\parallel}^2\cos^2\theta}}.$$

In this case the constant $E_0^{(j)}$ determines the initial amplitude of the field in the plane $z = z_0$.

Equations (3.6)–(3.13) have a clear physical meaning. They correspond to the adiabatic regime of the wave propagation. These equations can be considered as generalization for the case of the oblique incidence of the well known Mauguin solution [21]. Propagating between planes $z = z_0$ and z the normal wave with index j gains the phase $\int_{z_0}^{z} k_z^{(j)}(\mathbf{q},z')dz'$. As long as vectors $\mathbf{e}^{(j)}(\mathbf{q},z_0)$ and $\mathbf{e}^{(j)}(\mathbf{q},z)$ do not coincide the polarization vector rotates in the wave propagation process. The dependence of the amplitudes $A^{(j)}(\mathbf{q};z,z_0)$ on z in Eq. (3.12) is determined by the law of energy conservation for the wave propagating in the inhomogeneous medium without absorption. The wave vector $\mathbf{k}^{(j)}(\mathbf{q},z)$ in each fixed point of CLC is directed normally to the wave front. For the ordinary beam the wave vector $\mathbf{k}^{(1)} = \mathbf{k}^{(1)}(q)$ does not depend on z, whereas the modulus and the direction of the wave vector of the extraordinary beam $\mathbf{k}^{(2)} = \mathbf{k}^{(2)}(\mathbf{q},z)$ does depend on z. The directions of polarization vectors $\mathbf{e}^{(j)}(\mathbf{q},z)$ depend on z for both types of waves. For each vector $\mathbf{q}$ the wave vector $\mathbf{k}^{(j)}(\mathbf{q},z)$ is in the plane containing vectors $\mathbf{q}$ and $\mathbf{e}_z$ both for ordinary and extraordinary beams $\mathbf{e}_z$ is the unit vector along the z axis.

The tangent to the trajectory of the beam is parallel to the Poynting vector $\mathbf{S}$. Parametrizing the trajectory as $(\mathbf{r}_{\perp}(z),z)$ we can write

$$\frac{d\mathbf{r}_{\perp}(z)}{dz} = \frac{\mathbf{S}_{\perp}(z)}{S_z(z)}. \quad (3.14)$$

As far as $\delta_{(1)} = 0$, $\mathbf{S}^{(1)}(\mathbf{q},z) \parallel \mathbf{k}^{(1)}(q)$ and it does not depend on z. Therefore the trajectory of the ordinary beam is a straight line parallel to the wave vector $\mathbf{k}^{(1)}$.

In general $\delta_{(2)} \neq 0$ and as it follows from analysis of Eq. (3.11) the vector $\mathbf{S}^{(2)}(\mathbf{q},z)$ as a function of z does not belong to the same plane. Since our system is locally uniaxial, $\mathbf{S}^{(2)} \parallel \hat{\varepsilon}\mathbf{k}^{(2)}$. So

$$\frac{\mathbf{S}_{\perp}^{(2)}(z)}{S_z^{(2)}(z)} = \frac{(\hat{\varepsilon}(z)\mathbf{k}^{(2)}(z))_{\perp}}{(\hat{\varepsilon}(z)\mathbf{k}^{(2)}(z))_z} \quad (3.15)$$

and Eq. (3.14) for the trajectory of the extraordinary beam takes the form

$$\frac{d\mathbf{r}_{\perp}(z)}{dz} = \frac{\mathbf{n}^0(z)q\cos\phi(z)\varepsilon_a + \mathbf{q}\varepsilon_{\perp}}{k_z^{(2)}(\mathbf{q},z)\varepsilon_{\perp}}. \quad (3.16)$$

Integrating Eq. (3.16) we get the trajectory of the beam,

$$\mathbf{r}_{\perp}(z) = \frac{\varepsilon_a q}{\varepsilon_{\perp}}\int_0^z \frac{\mathbf{n}^0(z')\cos\phi(z')}{k_z^{(2)}(\mathbf{q},z')}dz' + \mathbf{q}\int_0^z \frac{dz'}{k_z^{(2)}(\mathbf{q},z')}. \quad (3.17)$$

A typical trajectory of the extraordinary beam calculated by Eq. (3.17) is shown in Fig. 2. Here the following parameters were used $\varepsilon_{\parallel} = 2.3$, $\varepsilon_a = 2.0$, the angle of incidence on the plane $z = 0$ is equal to $\pi/4$, and angle $\phi_0 = -\pi/4$. We take

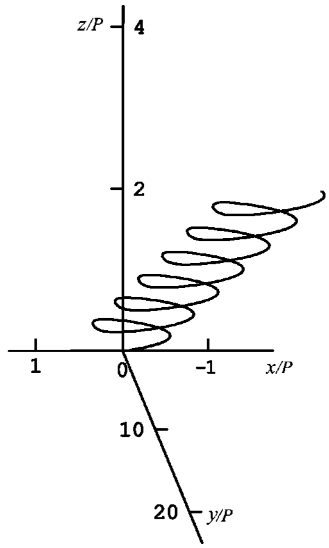


FIG. 2. A trajectory of the extraordinary beam in CLC. All distances are expressed in terms of P.

sufficiently large ε_a for better presentation only. One can see that the trajectory of the extraordinary beam is helixlike. Its pitch coincides with the pitch of CLC and the "diameter" of the helix grows with increasing of $\mathbf{q}$, i.e., with the increase of the incidence angle. The case $q=0$ corresponds to the propagation along the cholesteric axis and the trajectory becomes a straight line directed along the z axis. This case was investigated by Mauguin [21].

Now let us consider conditions when the waves (3.10) can penetrate into the CLC. For the ordinary wave $k_z^{(1)}$ is real provided the condition $q^2 \leqslant k_0^2\varepsilon_\perp$ is fulfilled. In this case the ordinary beam can penetrate to any depth in the z direction. If $q^2 > k_0^2\varepsilon_\perp$, the value of $k_z^{(1)}$ is imaginary and the ordinary wave cannot propagate inside the CLC.

For the extraordinary beam the conditions of propagation are more complex and depend on the relation between q and z parameters. In this case the following situations are possible:

(i) If $q^2 > k_0^2 \max(\varepsilon_\|, \varepsilon_\perp)$ then $k_z^{(2)}(\mathbf{q}, z)$ is imaginary for any z, and the wave cannot propagate inside CLC.

(ii) If $q^2 < k_0^2 \min(\varepsilon_\|, \varepsilon_\perp)$ then the value $k_z^{(2)}(\mathbf{q}, z)$ is real for any z and the wave can penetrate into CLC to any z.

(iii) If $k_0^2 \min(\varepsilon_\|, \varepsilon_\perp) \leqslant q^2 \leqslant k_0^2 \max(\varepsilon_\|, \varepsilon_\perp)$ then the extraordinary beam can propagate into the CLC in certain limits of z. The range of these values is determined by the inequality $\cos^2\phi(z) \leqslant \varepsilon_\perp(k_0^2\varepsilon_\| - q^2)/q^2\varepsilon_a$, for $\varepsilon_a > 0$, and the inequality $\cos^2\phi(z) \geqslant \varepsilon_\perp(k_0^2\varepsilon_\| - q^2)/q^2\varepsilon_a$ for $\varepsilon_a < 0$. Note that in the considered region of q the condition $0 \leqslant \varepsilon_\perp(k_0^2\varepsilon_\| - q^2)/q^2\varepsilon_a \leqslant 1$ is fulfilled.

So in the last case the capture of the extraordinary beam in CLC takes place [24,25]. From the physical point of view this effect implies that the beam starts to deviate and in the point $z = z^*(\mathbf{q})$ the component $k_z^{(2)}(\mathbf{q}, z)$ turns to zero changing then its sign. This effect in some aspect is similar to total reflection from a surface inside the medium. Since the refractive index is a periodical function of z such a beam would reflect alternately from two planes normal to the z axis. It means that a plane wave channel is formed and inside this channel the extraordinary beam propagates at a large distance along $\mathbf{r}_\perp$ remaining within one period in z. The projection of the ordinary and extraordinary beams and formation of the waveguide propagation are shown in Fig. 3. The curves are calculated for $\varepsilon_\| = 2.86$, $\varepsilon_\perp = 2.28$, $\phi_0 = 0$, and for different angles of incidence χ_0 on the plane $z=0$. These permittivities were used in experiments of Refs. [30,31]. For the ordinary beam $\chi_0 = 63.2°$ (1a) and $\chi_0 = 85.0°$ (1b), for the extraordinary beam the angle $\chi_0 = 63.2°$ (2a) corresponds to propagation outside of the wave guide channel and for angles $\chi_0 = 63.4°$ (2b) and $\chi_0 = 70.0°$ (2c) the wave guide regime takes place. Here the y axis is directed along the $\mathbf{q}$ vector.

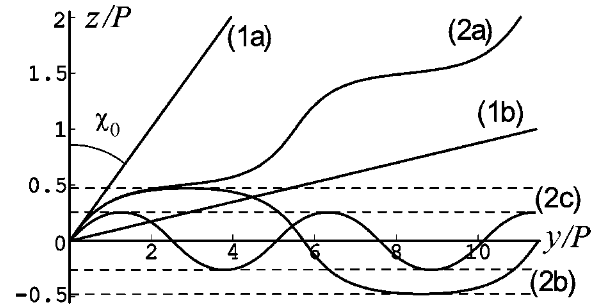


FIG. 3. The beam trajectories in CLC. Projections on the $z, \mathbf{k}_\perp$ plane. All distances are expressed in terms of P.

The trajectory of the ordinary beam is a straight line for an arbitrary angle of incidence. For extraordinary beams with no wave guide regime the trajectory is helixlike (Fig. 2). For extraordinary beams captured into a wave guide channel the trajectory is nonplane also and its form can be calculated by Eq. (3.17) with an additional condition that the component $k_z(\mathbf{q}, z')$ changes its sign in the turning points. The width of the wave guide channel depends on the incidence angle and varies within the limits $(0, P)$. The widest channel is restricted by planes with minimal values of the refractive index, $n_{(2)}(z) = \sqrt{\varepsilon_\perp}$. The beam capture starts with the angle of incidence,

$$\chi_0^* = \arcsin \frac{\sqrt{\varepsilon_\perp}}{\sqrt{\varepsilon_\| - \varepsilon_a \cos^2\phi_i}}, \tag{3.18}$$

where ϕ_i is the angle between $\mathbf{q}$ and $\mathbf{n}^0$ vectors in the plane $z=0$. For parameters $\varepsilon_\| = 2.86$, $\varepsilon_\perp = 2.28$ used in Fig. 3 the angle $\chi_0^* \approx 63.3°$. The effect of the extraordinary beam return was observed experimentally [30,31].

IV. FIELD OF THE POINT SOURCE

As far as our medium is homogeneous in the xy plane it is suitable to complete the transverse Fourier transformation,

$$f(\mathbf{r}) = \int \frac{d\mathbf{q}}{(2\pi)^2} f(\mathbf{q}, z) e^{i\mathbf{q}\cdot\mathbf{r}_\perp},$$

$$f(\mathbf{q},z) = \int d\mathbf{r}_\perp f(\mathbf{r}) e^{-i\mathbf{q}\cdot\mathbf{r}_\perp}. \quad (4.1)$$

The corresponding problem (2.7) for the field of the point source $\hat{T}^0(\mathbf{q};z,z_1)$ is reduced to a set of equations,

$$\hat{\mathcal{L}}(z)\hat{T}(\mathbf{q};z,z_1) = \delta(z-z_1)\hat{I}, \quad (4.2)$$

where

$$\mathcal{L}_{\alpha\beta}(z) = (e_{z\alpha}e_{z\beta} - \delta_{\alpha\beta})\frac{\partial^2}{\partial z^2} + i(q_\alpha e_{z\beta} + q_\beta e_{z\alpha})\frac{\partial}{\partial z} + (q^2\delta_{\alpha\beta} - q_\alpha q_\beta) - k_0^2\varepsilon_{\alpha\beta}(z)$$

is the linear differential operator of the second order. The first three terms of the operator $\hat{\mathcal{L}}$ correspond to operator curl curl in $(\mathbf{q},z)$ presentation. For convenience we consider here $\mathbf{q}$ as a three-dimensional vector with q_z component being equal to zero, $q_z=0$. Thus the calculation of the Green's function leads to solution of the system of nine differential equations with periodic coefficients. The principle of radiation determining the behavior of the solution at $z\to+\infty$ and $z\to-\infty$ is chosen as boundary conditions. The set of equations for the case $\Omega \gg 1$ was solved in Ref. [25], where the approach typical to the Sturm-Liouville problem was used. The solution of the inhomogeneous equation (4.2) is constructed as the superposition of solutions of the corresponding homogeneous equation (3.10) in the regions $z>z_1$ and $z<z_1$. The coefficients of the superposition are chosen so as to ensure the corresponding singularity in the right hand side of Eq. (4.2). The result has the form [25]

$$\hat{T}^0(\mathbf{q};z,z_1) = \hat{T}^{(1)}(\mathbf{q};z,z_1) + \hat{T}^{(2)}(\mathbf{q};z,z_1) + \hat{T}^{(3)}(\mathbf{q};z,z_1), \quad (4.3)$$

where

$$T^{(j)}_{\alpha\beta}(\mathbf{q};z,z_1) = \frac{i}{2k_0}B^{(j)}(\mathbf{q},z)B^{(j)}(\mathbf{q},z_1)e^{(j)}_\alpha(\mathbf{q},z) \times e^{(j)}_\beta(\mathbf{q},z_1)\exp\left(i\left|\int_{z_1}^{z} k_z^{(j)}(\mathbf{q},z')dz'\right|\right), \quad (4.4)$$

for $j=1$, 2, and

$$T^{(3)}_{\alpha\beta}(\mathbf{q};z,z_1) = -\frac{e_{z\alpha}e_{z\beta}}{k_0^2\varepsilon_\perp}\delta(z-z_1). \quad (4.5)$$

It can be shown that the solution (4.3) satisfies the Eq. (4.2) and the radiation condition. The first and the second terms of Eq. (4.3) contain oscillating factors and determine the asymptotics of the Green's function in the far zone. The third term is a short-range one and it contributes to the static field of the dipole only. In what follows this term is omitted.

In order to get the Green's function in the coordinate presentation it is necessary to complete inverse two-dimensional Fourier transformation. The integrals can be calculated by the stationary phase method. For this purpose it is necessary to find the stationary point $\mathbf{q}_{st}$ and then to expand the exponent in its vicinity as a Taylor series over $\mathbf{q}-\mathbf{q}_{st}$ up to terms of the second order. For the first term (4.3) the stationary point is determined by the condition

$$\nabla_{\mathbf{q}}[k_z^{(1)}(q)|z-z_1| + \mathbf{q}\cdot(\mathbf{r}_\perp - \mathbf{r}_{1\perp})] = 0. \quad (4.6)$$

Solving Eq. (4.6) we get

$$\mathbf{q}_{st}^{(1)} = \sqrt{\varepsilon_\perp}k_0\frac{\mathbf{r}_\perp - \mathbf{r}_{1\perp}}{|\mathbf{r}-\mathbf{r}_1|}. \quad (4.7)$$

The exponent in this term is equal to $i\Psi_1$ where

$$\Psi_1 = k_z^{(1)}(q_{st}^{(1)})|z-z_1| + \mathbf{q}_{st}^{(1)}\cdot(\mathbf{r}_\perp - \mathbf{r}_{1\perp}) = \sqrt{\varepsilon_\perp}k_0|\mathbf{r}-\mathbf{r}_1|. \quad (4.8)$$

For the second term of Eq. (4.3) the equation for the stationary point has the form

$$\mathbf{r}_\perp - \mathbf{r}_{1\perp} = \frac{1}{\varepsilon_\perp}\left|\int_{z_1}^{z}\frac{\hat{\varepsilon}^\perp(z')\mathbf{q}_{st}^{(2)}dz'}{k_z^{(2)}(\mathbf{q}_{st}^{(2)},z')}\right|, \quad (4.9)$$

where $\varepsilon^\perp_{kl}(z)=\varepsilon_{kl}(z)$, $(k,l=1,2)$ is the transverse projection of $\hat{\varepsilon}(z)$ on the xy plane. The exponent of the second term will be equal to $i\Psi_2$ where

$$\Psi_2 = \left|\int_{z_1}^{z} dz' k_z^{(2)}(\mathbf{q}_{st}^{(2)},z')\right| + \mathbf{q}_{st}^{(2)}\cdot(\mathbf{r}_\perp - \mathbf{r}_{1\perp}). \quad (4.10)$$

Thus we get the Green's function in the far zone

$$T_{\alpha\beta}(\mathbf{r},\mathbf{r}_1) = \frac{\exp(i\Psi_1)}{4\pi|\mathbf{r}-\mathbf{r}_1|}e_\alpha^{(1)}(\mathbf{q}_{st}^{(1)},z)e_\beta^{(1)}(\mathbf{q}_{st}^{(1)},z_1) + \frac{\exp(i\Psi_2)}{4\pi|\mathbf{r}-\mathbf{r}_1|}\frac{B^{(2)}(\mathbf{q}_{st}^{(2)},z)B^{(2)}(\mathbf{q}_{st}^{(2)},z_1)}{\sqrt{\det\hat{D}(\mathbf{q}_{st}^{(2)};\mathbf{r},\mathbf{r}_1)}} \times e_\alpha^{(2)}(\mathbf{q}_{st}^{(2)},z)e_\beta^{(2)}(\mathbf{q}_{st}^{(2)},z_1), \quad (4.11)$$

where

$$D_{kl}(\mathbf{q};\mathbf{r},\mathbf{r}_1) = -\frac{k_0}{|\mathbf{r}-\mathbf{r}_1|\sqrt{\varepsilon_\perp}}\left|\int_{z_1}^{z}\left[\frac{\varepsilon_{kl}^\perp}{[\varepsilon_\parallel\varepsilon_\perp k_0^2 - (\mathbf{q}\hat{\varepsilon}^\perp\mathbf{q})]^{1/2}} + \frac{(\hat{\varepsilon}^\perp\mathbf{q})_k(\hat{\varepsilon}^\perp\mathbf{q})_l}{[\varepsilon_\parallel\varepsilon_\perp k_0^2 - (\mathbf{q}\hat{\varepsilon}^\perp\mathbf{q})]^{3/2}}\right]dz'\right|. \quad (4.12)$$

According to Eq. (4.11) the Green's function decreases with the distance r as $1/r$. This behavior is valid if the phase (4.10) is real. The effect of the wave guide channel described in Sec. III appears for the Green's function too. For the second term of the Green's function (4.11) the wave guide regime takes place similar to the extraordinary normal wave. For waves captured in a wave guide channel Eq. (4.11) should be modified. In order to calculate the Green's function inside the channel it is necessary to sum the waves at a given point after various numbers of reflections. Here we do not study the problem of calculating the Green's function inside the wave channel but we only perform the qualitative analysis of the effect. First we obtain the decay law of the Green's function. As far as all waves hitting in the channel are being in plane layer for any distance $|\mathbf{r}_\perp - \mathbf{r}_{1\perp}|$, the wave energy inside the channel decreases as $|\mathbf{r}-\mathbf{r}_1|^{-1} \approx |\mathbf{r}_\perp$

$-\mathbf{r}_{1\perp}|^{-1}$. Hence the amplitude of the field decays as $|\mathbf{r}-\mathbf{r}_1|^{-1/2}$ instead of the usual behavior $|\mathbf{r}-\mathbf{r}_1|^{-1}$.

Let us estimate the fraction of the energy C_t of the extraordinary wave outgoing in the channel from the pointlike source situated in the origin of the coordinate frame [see Fig. 3, beams (2b) and (2c)]. This value is of the order of the fraction of the solid angle θ_t forming by the beams outgoing into the channel to the total solid angle 4π. For positive directions, $z>0$ the beams in the channel are radiated in the range of angles $\chi_0^*\leqslant\chi\leqslant\pi/2$ where the minimal angle χ_0^* is determined by Eq. (3.18). The solid angle θ_t has the form

$$\theta_t = 2\int_0^{2\pi} d\phi_i\left(\int_{\chi_0^*}^{\pi/2}\sin\chi\, d\chi\right). \tag{4.13}$$

Here factor 2 is introduced in order to take into account beams propagating in the negative direction, $z<0$. Calculating integrals (4.13) we get

$$C_t \sim \frac{\theta_t}{4\pi} = \frac{2}{\pi}\arctan\frac{\sqrt{\varepsilon_a}}{\sqrt{\varepsilon_\perp}}. \tag{4.14}$$

For $\varepsilon_\perp=2.28$ and $\varepsilon_\parallel=2.86$ used in calculation, Fig. 3, the fraction of the energy outgoing into the wave guide channel is $C_t\approx 0.30$.

Let us analyze the limit $q_0\to 0$ in Eq. (4.4) corresponding to the uniaxial homogeneous medium, in particular to nematic liquid crystal in our case. The values $\mathbf{n}^0(z)$, $k_z^{(j)}(\mathbf{q},z)$, $B^{(j)}(\mathbf{q},z)$, $\mathbf{e}^{(j)}(\mathbf{q},z)$ in this limit do not depend on z: $\mathbf{n}^0(z)=\mathbf{n}^0$, $k_z^{(j)}(\mathbf{q},z)=k_z^{(j)}(\mathbf{q})$, $B^{(j)}(\mathbf{q},z)=B^{(j)}(\mathbf{q})$, $\mathbf{e}^{(j)}(\mathbf{q},z)=\mathbf{e}^{(j)}(\mathbf{q})$ whereas the function $\hat{T}^{(j)}(\mathbf{q};z,z_1)$ depends on the difference of the spatial coordinates, $\hat{T}^{(j)}(\mathbf{q};z,z_1)=\hat{T}^{(j)}(\mathbf{q};z-z_1)$ only. Equation (4.4) in this limit is

$$T_{\alpha\beta}^{(j)}(\mathbf{q};z-z_1) = \frac{i}{2k_0}B^{(j)2}(\mathbf{q})e_\alpha^{(j)}(\mathbf{q})e_\beta^{(j)}(\mathbf{q})e^{ik_z^{(j)}(\mathbf{q})|z-z_1|}. \tag{4.15}$$

In homogeneous uniaxial media the three-dimensional Fourier transform of the field of the point source has the form [32]

$$T_{\alpha\beta}^0(\mathbf{Q}) = \frac{1}{k_0^2}\left(\sum_{j=1,2}\frac{e_{(j)\alpha}(\mathbf{Q})e_{(j)\beta}(\mathbf{Q})}{\mathbf{e}_{(j)}(\mathbf{Q})\hat{\varepsilon}^0\mathbf{e}_{(j)}(\mathbf{Q})}\frac{k_{(j)}^2(\mathbf{Q})}{Q^2-k_{(j)}^2(\mathbf{Q})-i0} - \frac{Q_\alpha Q_\beta}{\mathbf{Q}\hat{\varepsilon}^0\mathbf{Q}}\right). \tag{4.16}$$

Here $\mathbf{Q}$ is the three-dimensional wave vector, $\mathbf{e}_{(j)}$ are the polarization vectors, and $k_{(j)}$ are the wave vectors of the ordinary and extraordinary plane waves propagating in homogeneous uniaxial media. The corresponding wave numbers and polarization vectors of the ordinary and extraordinary waves have the form (3.6) and (3.8) with the substitution of $\mathbf{k}(z)$ into $\mathbf{Q}$:

$$k_{(j)}(\mathbf{Q}) = k^{(j)}(\mathbf{Q}), \quad \mathbf{e}_{(j)}(\mathbf{Q}) = \mathbf{e}^{(j)}(\mathbf{Q}). \tag{4.17}$$

Let us present the wave vector $\mathbf{Q}$ in the form $\mathbf{Q}=(\mathbf{q};q_z)$ and fulfill the inverse Fourier transformation over q_z in Eq. (4.16),

$$T_{\alpha\beta}^0(\mathbf{q};z) = \int_{-\infty}^{\infty}\frac{dq_z}{2\pi}T_{\alpha\beta}^0(\mathbf{q};q_z)e^{iq_z z}. \tag{4.18}$$

For $z\gg\lambda$ the principal contribution to asymptotics of this integral is determined by the poles of the first two terms in Eq. (4.16). These poles can be found by solving the dispersion equations,

$$q^2 + q_z^2 - k_{(j)}^2(\mathbf{q},q_z) = 0, \tag{4.19}$$

$j=1$, 2. Completing one-dimensional Fourier transformation over q_z we get from Eq. (4.16)

$$T_{\alpha\beta}^0(\mathbf{q};z) = \frac{i}{k_0^2}\sum_{j=1,2}k_{(j)}^2\left(2q_z^{(j)} - \left.\frac{\partial k_{(j)}^2}{\partial q_z}\right|_{q_z=q_z^{(j)}}\right)^{-1} \times \frac{e_{(j)\alpha}e_{(j)\beta}}{\mathbf{e}_{(j)}\hat{\varepsilon}^0\mathbf{e}_{(j)}}\exp(iq_z^{(j)}|z|), \tag{4.20}$$

where $\pm q_z^{(j)}(\mathbf{q})$ are two solutions of Eq. (4.19); values of $\mathbf{e}_{(j)}$ and $k_{(j)}$ are calculated for the wave vector $\mathbf{Q}^{(j)}(\mathbf{q})\equiv(\mathbf{q},q_z^{(j)}(\mathbf{q}))$.

Substituting expressions (4.17) for $k_{(j)}^2(\mathbf{Q})$ into Eq. (4.19) and solving the obtained equations for $j=1$, 2 with respect to q_z we get in both cases that $q_z^{(j)}=k_z^{(j)}(\mathbf{q})$ where $k_z^{(j)}(\mathbf{q})$ is determined in Eq. (3.9). Note that for $j=2$ it is necessary to take into account that $\mathbf{Q}\cdot\mathbf{n}^0=\mathbf{q}\cdot\mathbf{n}^0$. Thus we have $\mathbf{Q}^{(j)}(\mathbf{q})=(\mathbf{q},k_z^{(j)}(\mathbf{q}))=\mathbf{k}^{(j)}(\mathbf{q})$. Using Eq. (4.17) it is not difficult to verify that values of $\mathbf{e}_{(j)}(\mathbf{k}^{(j)}(\mathbf{q}))$ and $k_{(j)}(\mathbf{k}^{(j)}(\mathbf{q}))$ coincide with $\mathbf{e}^{(j)}(\mathbf{q})$ and $k^{(j)}(\mathbf{q})$, respectively, in Eqs. (3.8) and (3.9) for $q_0=0$. Also it is easy to verify the identity

$$\frac{k_z^{(j)}(\mathbf{q})B^{(j)2}(\mathbf{q})}{k_0} = \frac{k^{(j)2}(\mathbf{q})}{k_0^2\mathbf{e}^{(j)}(\mathbf{q})\hat{\varepsilon}^0\mathbf{e}^{(j)}(\mathbf{q})} \times\left(q_z - \frac{1}{2}\left.\frac{\partial k_{(j)}^2(\mathbf{q},q_z)}{\partial q_z}\right|_{q_z=k_z^{(j)}(\mathbf{q})}\right)^{-1} \tag{4.21}$$

for both cases, $j=1$, 2. As a result Eq. (4.20) coincides with Eqs. (4.3) and (4.15).

V. GENERAL THEORY OF THE SINGLE LIGHT SCATTERING IN THE STRATIFIED MEDIA

In our medium the normal waves are not plane, the Green's function has a complicated structure and the correlation function of the permittivity fluctuations $\hat{\mathcal{G}}(\mathbf{r}_1,\mathbf{r}_2)$ depends not only on the difference of the spatial coordinates but also on their values separately. Therefore the intensity of the single scattering is not proportional to the three-dimensional Fourier transformation of the permittivity fluctuations $\delta\varepsilon$ for the scattering vector $\mathbf{k}^{(s)}-\mathbf{k}^{(i)}$ [see Eq. (B1)].

Another problem results from Eq. (2.8) which describes the scattered field inside the medium only, whereas in an

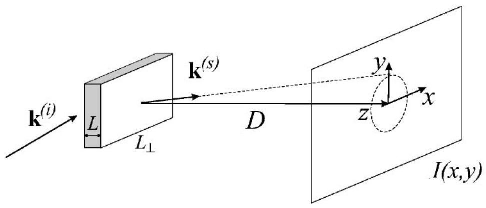


FIG. 4. Geometry of light scattering. Here D is the distance between the slab and the screen.

experiment the scattered light is usually measured outside the medium. For homogeneous systems this problem is solved in the following way. The scattering volume is assumed to be imbedded into a homogeneous medium with permittivity ε_0 in order to avoid the problem of refraction near the boundary of the specimen (generally the refraction at the boundary could be taken into account). As far as in the homogeneous medium the incident wave is plane and the scattered wave in the far zone inside the medium can be considered as the quasiplane the refraction problem can be treated with the use of the Fresnel formulas. In this case there arises a problem of recalculation of solid angles from the incident to the refracted beam. This problem was considered in Refs. [33] for general case of anisotropic media.

There is a fundamental difference between the scattering medium with the periodic inhomogeneities and the homogeneous environment. The normal waves and the Green's function inside and outside the scattering volume are essentially different. In particular the incident and the scattered waves can be considered as plane waves outside the medium only, so the effect of the boundary is important.

In order to overcome this obstacle for the scattering problem in the medium with one-dimensional regular inhomogeneities we suggest the Kirchhoff method. The problem is solved in three stages: (i) The scattered field is calculated inside the medium in the $(\mathbf{q},z)$ representation. (ii) We recalculate the scattered field in the boundary outside the medium into that in the outside space. (iii) The coordinate representation of the field in the outside area is calculated on the basis of its value in the boundary outside the medium in $(\mathbf{q},z)$ representation.

In what follows we assume that the scattering volume is the plane layer, $0 \leqslant z \leqslant L$, with a large transverse size $L_\perp \gg L$ (Fig. 4). The incident plane wave starts from $z=-\infty$ and the scattered field is recorded in the region $z > L$, i.e., in the positive half space. The latter is not essential as far as scattering in the negative half space, $z<0$, can be considered in a similar way. Let the incident field is a plane wave with the wave vector $\mathbf{k}^{(i)}$. The scattered wave has the wave vector $\mathbf{k}^{(s)}$ and is measured in the far zone.

Due to the identity $k_z^2+k_\perp^2=k_0^2\varepsilon_0$ which is valid outside the stratified medium the total wave vector $\mathbf{k}$ is determined by the component $\mathbf{k}_\perp$ and the sign of the k_z component. Therefore it is sufficient to define the vector $\mathbf{k}_\perp^{(i)}$ and direction of the incident wave, positive or negative, with respect to z. The latter is valid for the wave vector $\mathbf{k}_\perp^{(s)}$.

Below indices "*in*" and "*out*" refer to the values inside and outside of the inhomogeneous medium, respectively.

A. Kirchhoff method

Let us consider an arbitrary inhomogeneous specimen Γ bounded by a closed surface Σ situated outside in a homogeneous medium. The electromagnetic field outside the inhomogeneous specimen, $\mathbf{E}(\mathbf{r})=\mathbf{E}_{\text{out}}(\mathbf{r})$, satisfies the wave equation

$$(\text{curl curl} - k_0^2\varepsilon_0)\mathbf{E}(\mathbf{r}) = 0. \tag{5.1}$$

It is easy to notice that the system of the three equations (5.1) is equivalent to the system

$$\begin{cases}(\Delta + k_0^2\varepsilon_0)\mathbf{E}(\mathbf{r}) = 0, \\ \text{div}\,\mathbf{E}(\mathbf{r}) = 0.\end{cases} \tag{5.2}$$

The first equation (5.2) implies that each vector component satisfy the scalar Helmholtz equation and the second equation yields the additional condition div $\mathbf{E}=0$ since the electromagnetic field is transversal.

The Green's function $T(\mathbf{r},\mathbf{r}')=T_{\text{out}}(\mathbf{r},\mathbf{r}')$, $\mathbf{r},\mathbf{r}' \notin \Gamma$, satisfies the equation

$$(\Delta + k_0^2\varepsilon_0)T(\mathbf{r},\mathbf{r}') = -\delta(\mathbf{r}-\mathbf{r}'). \tag{5.3}$$

Equation (5.3) does not define the function $T(\mathbf{r},\mathbf{r}')$ uniquely and additional boundary conditions are required. If we take $T|_\Sigma=0$ as the boundary condition then the field $E_\alpha(\mathbf{r})$ in the observation point may be expressed in terms of the field $E_\alpha(\mathbf{r}')$ on the surface Σ. According to the Kirchhoff-Helmholtz integral theorem [34],

$$E_\alpha(\mathbf{r}) = -\int_\Sigma d^2r' E_\alpha(\mathbf{r}')\nabla_{\mathbf{r}'}T(\mathbf{r},\mathbf{r}')\cdot\mathbf{s}(\mathbf{r}'), \tag{5.4}$$

where $\mathbf{s}(\mathbf{r}')$ is the external normal to the surface Σ in the $\mathbf{r}'$ point.

The expression for the Green's function satisfying the condition $T|_\Sigma=0$ depends on the form of the specimen. For simplicity we shall consider the surface Σ as a piece of the plane $z=L$ with a large transverse size $L_\perp$ closed by a large hemisphere. If the Green's function $T(\mathbf{r},\mathbf{r}')$ satisfies the radiation condition in the infinity then the contribution of the hemisphere to integral (5.4) tends to zero with increasing of its radius. In this case the boundary condition $T|_\Sigma=0$ in our geometry is reduced to $T|_{z=L}=0$, and using the mirror image method we get

$$T(\mathbf{r},\mathbf{r}') = \frac{1}{4\pi}\left(\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} - \frac{e^{ik|\mathbf{r}-\mathbf{r}_1'|}}{|\mathbf{r}-\mathbf{r}_1'|}\right), \tag{5.5}$$

where $\mathbf{r}_1'$ is the mirror image of $\mathbf{r}'$ point with respect to the boundary plane $z=L$.

Let us suppose that the field is measured in the point $\mathbf{r}=(\mathbf{r}_\perp,z)$, $z-L \gg L_\perp$. Then in both terms of Eq. (5.5) we can use the plane wave approximation of the form

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \approx \frac{e^{ikr}}{r}e^{-i\mathbf{k}^{(s)}\cdot\mathbf{r}'}. \tag{5.6}$$

As far as in our geometry $\mathbf{s}(\mathbf{r}')\cdot\nabla_{\mathbf{r}'}=-\partial/\partial z'$, we get from Eq. (5.4)

$$\mathbf{E}(\mathbf{r}) = \frac{-ik_0\sqrt{\varepsilon_0}}{2\pi}\frac{e^{ikr}}{r}\frac{z}{r}e^{-ik_z^{(s)}L}\hat{P}(\mathbf{r})\mathbf{E}(\mathbf{k}_\perp^{(s)},\mathbf{L}). \quad (5.7)$$

In order to fulfill the condition div $\mathbf{E}=0$ we multiplied the field by the projector

$$\hat{P}(\mathbf{r}) = \hat{I} - \frac{\mathbf{r}\otimes\mathbf{r}}{r^2},$$

providing the field to be transversal in the far zone. This way we get the vector analog of the Kirchhoff formula.[1]

Now due to relation $e_\alpha^{(s)}P_{\alpha\beta}=e_\beta^{(s)}$ we obtain the intensity of the scattered field with polarization $\mathbf{e}^{(s)}$,

$$I = \frac{\sqrt{\varepsilon_0}c}{8\pi}\frac{k_0^2\varepsilon_0}{4\pi^2}\frac{1}{r^2}\left(\frac{z}{r}\right)^2\langle|\mathbf{e}^{(s)}\cdot\mathbf{E}_{\text{out}}^{(s)}(\mathbf{k}_\perp^{(s)},\mathbf{L})|^2\rangle. \quad (5.8)$$

Thus the scattered field in the $(\mathbf{q},z)$ representation in the boundary outside the media, $\mathbf{E}_{\text{out}}^{(s)}$ is required for the following calculations.

B. Recalculation of the field in the specimen boundaries

The relations between field components inside and outside the medium are determined by the boundary conditions of electrodynamics. According to these conditions the transverse components of the wave vectors and the fields do not change when the waves pass through the boundaries,

$$\mathbf{E}_{\text{out}\perp}^{(i)}(\mathbf{k}_\perp^{(i)},0) = \mathbf{E}_{\text{in}\perp}^{(i)}(\mathbf{k}_\perp^{(i)},0),$$

$$\mathbf{E}_{\text{out}\perp}^{(s)}(\mathbf{k}_\perp^{(s)},L) = \mathbf{E}_{\text{in}\perp}^{(s)}(\mathbf{k}_\perp^{(s)},L). \quad (5.9)$$

The z components of the fields could be obtained from the condition for the induction vector div $\mathbf{D}=0$. This condition gives

$$D_{\text{out }z}^{(i)}(\mathbf{k}_\perp^{(i)},0) = D_{\text{in }z}^{(i)}(\mathbf{k}_\perp^{(i)},0),$$

$$D_{\text{out }z}^{(s)}(\mathbf{k}_\perp^{(s)},L) = D_{\text{in }z}^{(s)}(\mathbf{k}_\perp^{(s)},L). \quad (5.10)$$

So linear relations between the fields $\mathbf{E}_{\text{in}}$ and $\mathbf{E}_{\text{out}}$ are valid at the boundaries inside and outside the specimen

$$\mathbf{E}_{\text{in}}^{(i)}(\mathbf{k}_\perp^{(i)},0) = \hat{M}^{\text{out}\to\text{in}}(\mathbf{k}_\perp^{(i)},0)\mathbf{E}_{\text{out}}^{(i)}(\mathbf{k}_\perp^{(i)},0),$$

$$\mathbf{E}_{\text{out}}^{(s)}(\mathbf{k}_\perp^{(s)},L) = \hat{M}^{\text{in}\to\text{out}}(\mathbf{k}_\perp^{(s)},L)\mathbf{E}_{\text{in}}^{(s)}(\mathbf{k}_\perp^{(s)},L). \quad (5.11)$$

Here $\hat{M}^{\text{out}\to\text{in}}$ and $\hat{M}^{\text{in}\to\text{out}}$ are transformation matrices which can be calculated from Eqs. (5.9) and (5.10). As far as the director component $n_z^0=0$ these matrices do not depend on $\mathbf{k}_\perp$ and we get

$$\hat{M}^{\text{out}\to\text{in}} = \widehat{\text{diag}}(1,1,\varepsilon_0/\varepsilon_\perp),$$

$$\hat{M}^{\text{in}\to\text{out}} = (\hat{M}^{\text{out}\to\text{in}})^{-1}, \quad (5.12)$$

where diag denotes the diagonal matrix.

VI. SCATTERING OF LIGHT IN CLC

For stratified media with boundaries parallel to the layers the wave inside the medium has the form

$$\mathbf{E}_{\text{in}}^{(i)}(\mathbf{r}) = \boldsymbol{\mathcal{E}}^{(i)}(\mathbf{k}_\perp^{(i)},z)e^{i\mathbf{k}_\perp^{(i)}\cdot\mathbf{r}_\perp}, \quad (6.1)$$

where $\boldsymbol{\mathcal{E}}^{(i)}(\mathbf{k}_\perp^{(i)},z)$ is determined by the properties of the stratified medium and also by polarization and amplitude of the incident wave. In case $P\gg\lambda$ the function $\boldsymbol{\mathcal{E}}^{(i)}$ is determined by Eqs. (3.10) and (3.12).

From Eqs. (2.8) and (6.1) we obtain the scattered field $\mathbf{E}_{\text{in}}^{(s)}(\mathbf{k}_\perp^{(s)},L)$ at the boundary $z=L$ inside the specimen

$$\mathbf{E}_{\text{in}}^{(s)}(\mathbf{k}_\perp^{(s)},L) = k_0^2\int_0^L dz'\,\hat{T}^0(\mathbf{k}_\perp^{(s)};L,z')$$
$$\times\,\delta\hat{\varepsilon}(\mathbf{k}_\perp^{(s)}-\mathbf{k}_\perp^{(i)},z')\boldsymbol{\mathcal{E}}^{(i)}(\mathbf{k}_\perp^{(i)},z'). \quad (6.2)$$

From Eqs. (5.11) and (6.2) and relation $\langle\delta\hat{\varepsilon}(\mathbf{k}_\perp,z)\otimes\delta\hat{\varepsilon}^*(\mathbf{k}_\perp,z')\rangle=S_\perp\hat{\mathcal{G}}(\mathbf{k}_\perp;z,z')$ we obtain the intensity of the single light scattering outside the specimen (5.8) in the form

$$I = \frac{\sqrt{\varepsilon_0}c}{8\pi}\frac{k_0^6\varepsilon_0}{4\pi^2}\frac{S_\perp}{r^2}\left(\frac{z}{r}\right)^2 e_\alpha^{(s)}e_\gamma^{(s)}M_{\alpha\beta}^{\text{in}\to\text{out}}M_{\gamma\delta}^{\text{in}\to\text{out}}$$
$$\times\int_0^L dz_1\int_0^L dz_2 T_{\beta\rho}^0(\mathbf{k}_\perp^{(s)};L,z_1)T_{\delta\varphi}^{0*}(\mathbf{k}_\perp^{(s)};L,z_2)$$
$$\times\,\mathcal{G}_{\rho\nu\varphi\mu}(\mathbf{k}_\perp^{(s)}-\mathbf{k}_\perp^{(i)};z_1,z_2)\mathcal{E}_\nu^{(i)}(\mathbf{k}_\perp^{(i)},z_1)\mathcal{E}_\mu^{(i)*}(\mathbf{k}_\perp^{(i)},z_2), \quad (6.3)$$

where $S_\perp$ is the cross-section area of the specimen.

We restrict our treatment to the case when the polarization of the incident light inside the medium has only one of two possible types of waves $\mathbf{E}_{\text{in}}^{(i)}(\mathbf{r})$, Eq. (3.10). Otherwise the summation over (i) should be performed for the field inside the medium. In a similar way the scattered light outside the medium corresponds only to one type of the scattered wave inside the medium, $\mathbf{E}_{\text{in}}^{(s)}(\mathbf{r})$. So in what follows it is possible to omit the summation over (s) for the field inside the medium. Thus indices (i) and (s) take values 1, 2 dependent on types of the incident and scattered waves.

A. Light scattering intensity in CLC with large pitch

Expression for the scattering intensity (6.3) contains the conjugate couples of fields and Green's functions. Substituting expressions for the incident field, Eqs. (3.10), and the Green's function, Eq. (4.4), into Eq. (6.3) we get the intensity of scattering,

[1]It is known [34] that direct translation of the scalar Kirchhoff formula to the vector problem leads to violation of the relation div $\mathbf{E}=0$, i.e., the field becomes nontransversal. For elimination of this contradiction it is possible to apply the Kirchhoff-Kottler relation (see, e.g., Ref. [35]). However, this effect in the far zone is of the order of $\lambda/L_\perp\ll1$. Therefore we shall use the vector equation (5.7) where the projector $P_{\alpha\beta}(\mathbf{r})$ makes our field transversal. In the far zone this approach is similar to the Kirchhoff-Kottler method.

$$I = J_0\left(\frac{z}{r}\right)^2 \frac{B^{(s)2}(\mathbf{k}_\perp^{(s)},L)}{B^{(i)2}(\mathbf{k}_\perp^{(i)},0)} \frac{1}{L}\int_0^L dz_1 \int_0^L dz_2$$
$$\times \exp\left[i\int_{z_1}^{z_2} q_z^{(sc)}(z')dz' \right] B^{(i)}(\mathbf{k}_\perp^{(i)},z_1)B^{(i)}(\mathbf{k}_\perp^{(i)},z_2)$$
$$\times B^{(s)}(\mathbf{k}_\perp^{(s)},z_1)B^{(s)}(\mathbf{k}_\perp^{(s)},z_2)e_\rho^{(s)}(\mathbf{k}_\perp^{(s)},z_1)e_\varphi^{(s)}(\mathbf{k}_\perp^{(s)},z_2)$$
$$\times \mathcal{G}_{\rho\nu\varphi\mu}(\mathbf{q}_\perp^{(sc)};z_1,z_2)e_\nu^{(i)}(\mathbf{k}_\perp^{(i)},z_1)e_\mu^{(i)}(\mathbf{k}_\perp^{(i)},z_2), \qquad (6.4)$$

where $\mathbf{q}_\perp^{(sc)}=\mathbf{k}_\perp^{(s)}-\mathbf{k}_\perp^{(i)}$, $q_z^{(sc)}(z)=k_z^{(s)}(\mathbf{k}_\perp^{(s)},z)-k_z^{(i)}(\mathbf{k}_\perp^{(i)},z)$,

$$J_0 = \frac{\sqrt{\varepsilon_0}cE_{0\text{ in}}^{(i)2}}{8\pi}\frac{k_0^4\varepsilon_0}{16\pi^2}\frac{V_{\text{sc}}}{r^2}e_\alpha^{(s)}e_\gamma^{(s)}M_{\alpha\beta}^{\text{in}\to\text{out}}M_{\gamma\delta}^{\text{in}\to\text{out}}$$
$$\times e_\beta^{(s)}(\mathbf{k}_\perp^{(s)},L)e_\delta^{(s)}(\mathbf{k}_\perp^{(s)},L),$$

and $V_{\text{sc}}=S_\perp L$ is the scattering volume. Using the first equation in Eq. (5.12) we can calculate the field $E_{0\text{ in}}^{(i)}$ inside the medium through the field $E_{0\text{ out}}^{(i)}$ outside the medium:

$$E_{0\text{ in}}^{(i)} = E_{0\text{ out}}^{(i)}\left[e_{\text{out}\perp}^{(i)2} + \frac{\varepsilon_0^2}{\varepsilon_\perp^2}e_{\text{outz}}^{(i)2}\right]^{-1/2},$$

where $\mathbf{e}_{\text{out}}^{(i)}$ is the polarization vector of the incident field outside the medium.

Integral (6.4) contains rapidly oscillating factor $\exp[i\int_{z_1}^{z_2}q_z^{(sc)}(z')dz']$. As far as the vicinity of the line $z_1=z_2$ yields the main contribution to the asymptotic behavior of the integral (6.4) it is convenient to introduce new variables $z_+=(z_1+z_2)/2$ and $z_-=z_2-z_1$. Expanding the phase function in series near the line $z_-=0$ up to the terms of the first order we have

$$\int_{z_+-z_-/2}^{z_++z_-/2} q_z^{(sc)}(z')dz' \approx q_z^{(sc)}(z_+)z_-. \qquad (6.5)$$

This approach is valid for $q_z^{(sc)}P \gg 1$. But as it follows from Appendix A the correlation function (A21) contains rapidly decaying factors $\exp[-q_\perp^{(sc)}|\int_{z_1}^{z_2}\mu_j(z)dz|]$. Therefore the approach is valid not only for $q_z^{(sc)}P\gg 1$ but also for $q_z^{(sc)}P \sim 1$, $q_\perp^{(sc)}P\gg 1$, i.e., finally for $q^{(sc)}P\gg 1$.

Functions $B^{(i)}(\mathbf{k}_\perp^{(i)},z)$, $B^{(s)}(\mathbf{k}_\perp^{(s)},z)$ and $e_\beta^{(s,i)}(\mathbf{k}_\perp^{(s,i)},z)$ vary slowly compared to the rapidly oscillating function $\exp[iq_z^{(sc)}(z_+)z_-]$. Therefore it is possible to substitute z_+ instead of z_1 and z_2 into these functions. We can expand the region of integration over the z_- variable within the limits $\pm\infty$ and for the correlation function we get the Fourier image $\hat{\mathcal{G}}(\mathbf{q}_\perp^{(sc)},q_z^{(sc)}(z_+),z_+)$.

Thus the intensity of light scattering has the form

$$I = J_0\left(\frac{z}{r}\right)^2 \frac{B^{(s)2}(\mathbf{k}_\perp^{(s)},L)}{B^{(i)2}(\mathbf{k}_\perp^{(i)},0)}\frac{1}{L}\int_0^L dz_+ B^{(i)2}(\mathbf{k}_\perp^{(i)},z_+)$$
$$\times B^{(s)2}(\mathbf{k}_\perp^{(s)},z_+)e_\rho^{(s)}(\mathbf{k}_\perp^{(s)},z_+)e_\varphi^{(s)}(\mathbf{k}_\perp^{(s)},z_+)$$
$$\times \mathcal{G}_{\rho\nu\varphi\mu}(\mathbf{q}^{(sc)}(z_+),z_+)e_\nu^{(i)}(\mathbf{k}_\perp^{(i)},z_+)e_\mu^{(i)}(\mathbf{k}_\perp^{(i)},z_+), \quad (6.6)$$

where $\mathbf{q}^{(sc)}(z)=(\mathbf{q}_\perp^{(sc)},q_z^{(sc)}(z))$.

Equation (6.6) describing the scattering intensity in the Born approximation in the far zone refers to the spatially inhomogeneous scattering medium and it is derived using the Kirchhoff method. We show in Appendix B that this equation is reduced to the standard expression of the scattering theory in the limit of the spatially homogeneous medium.

The correlation functions $\hat{g}(\mathbf{q}^{(sc)}(z),z)$ and $\hat{\mathcal{G}}(\mathbf{q}^{(sc)}(z),z)$ for the case $q_\perp^{(sc)}P\gg 1$ are considered in Appendix A. Substituting Eq. (A32) into Eq. (6.6) we get for the light scattering intensity

$$I \equiv I(\mathbf{e}^{(i)},\mathbf{e}^{(s)})$$
$$= J_0 k_B T\varepsilon_a^2\left(\frac{z}{r}\right)^2\frac{B^{(s)2}(\mathbf{k}_\perp^{(s)},L)}{B^{(i)2}(\mathbf{k}_\perp^{(i)},0)}$$
$$\times \frac{1}{L}\int_0^L dz\sum_{j=1,2}\frac{B^{(i)2}(\mathbf{k}_\perp^{(i)},z)B^{(s)2}(\mathbf{k}_\perp^{(s)},z)}{K_{jj}q^{(sc)2}+(K_{33}-K_{jj})(\mathbf{q}^{(sc)}\cdot\mathbf{n}^0)^2}$$
$$\times[(\mathbf{e}_j\cdot\mathbf{e}^{(i)})(\mathbf{n}^0\cdot\mathbf{e}^{(s)})+(\mathbf{e}_j\cdot\mathbf{e}^{(s)})(\mathbf{n}^0\cdot\mathbf{e}^{(i)})]^2, \qquad (6.7)$$

where $\mathbf{n}^0=\mathbf{n}^0(z)$, $\mathbf{e}^{(i)}=\mathbf{e}^{(i)}(\mathbf{k}_\perp^{(i)},z)$, $\mathbf{e}^{(s)}=\mathbf{e}^{(s)}(\mathbf{k}_\perp^{(s)},z)$, $\mathbf{e}_j=\mathbf{e}_j(\mathbf{q}^{(sc)},z)$, $\mathbf{q}^{(sc)}=\mathbf{q}^{(sc)}(z)$. Comparing the applicability conditions for Eqs. (6.6) and (A32) we finally get that Eq. (6.7) is valid for $q_\perp^{(sc)}P\gg 1$.

Note, that in our case the correlation function $\hat{\mathcal{G}}$, the wave vector $\mathbf{q}^{(sc)}$, and the amplitude factors $B^{(j)}$ depend on the z variable. According to Eq. (6.7) the intensity of scattering presents a sum of scattering intensities produced by the nematiclike layers of thickness dz. The director orientation $\mathbf{n}^0$ of each layer coincides with the specimen orientation in the z point. The scattering intensity from each layer has the form [see Eq. (B2)]

$$dI = \frac{I_0 k_0^4 dV_{\text{sc}}}{(4\pi)^2 r^2}e_\alpha^{(s)}e_\beta^{(s)}\mathcal{G}_{\alpha\nu\beta\mu}(\mathbf{q}^{(sc)})e_\nu^{(i)}e_\mu^{(i)},$$

where I_0 is the intensity of the incident light, $dV_{\text{sc}}=L_\perp dz$ is the layer volume. According to Eqs. (3.10) and (4.4) the factors $B^{(i)}$ and $B^{(s)}$ determine the amplitudes of the incident and scattered fields in each layer.

B. Basic scattering geometries

Let us analyze the light scattering intensities for various polarizations. In what follows we use the notations (o) and (e) for ordinary and extraordinary beams. In this system there exist four types of scattering, $(i)-(s)$.

The scattering of the $(o)-(o)$ type is absent since the polarization vector of the ordinary beam is perpendicular to the director, $\mathbf{n}^0\cdot\mathbf{e}^{(1)}=0$. So for the $(o)-(o)$ scattering it is valid that $\mathbf{n}^0\cdot\mathbf{e}^{(i)}=0$, $\mathbf{n}^0\cdot\mathbf{e}^{(s)}=0$ and hence the scattering intensity (6.7) goes to zero. This situation is similar to that for the nematic liquid crystal.

In the case of $(o)-(e)$ scattering there is only one nonzero term in brackets of Eq. (6.7). So we get

$$I(\mathbf{e}^{(1)},\mathbf{e}^{(2)}) = J_0 k_B T \varepsilon_a^2 \left(\frac{z}{r}\right)^2 B^{(2)2}(\mathbf{k}_\perp^{(s)},L) \times \frac{1}{L}\int_0^L dz B^{(2)2}(\mathbf{k}_\perp^{(s)},z)[\mathbf{n}^0(z)\cdot \mathbf{e}^{(2)}(\mathbf{k}_\perp^{(s)},z)]^2 \times \sum_{j=1,2} \frac{[\mathbf{e}_j(\mathbf{q}^{(sc)},z)\cdot \mathbf{e}^{(1)}(\mathbf{k}_\perp^{(i)},z)]^2}{K_{jj}q^{(sc)2}(z) + (K_{33}-K_{jj})[\mathbf{q}^{(sc)}(z)\cdot \mathbf{n}^0(z)]^2}. \tag{6.8}$$

Intensity of the $(e)-(o)$ scattering can be obtained from $(o)-(e)$ scattering intensity if we substitute $\mathbf{e}^{(1)} \rightleftharpoons \mathbf{e}^{(2)}$ and $\mathbf{k}^{(s)} \rightleftharpoons \mathbf{k}^{(i)}$.

For $(e)-(e)$ scattering both terms in brackets in Eq. (6.7) contribute in general to the intensity,

$$I(\mathbf{e}^{(2)},\mathbf{e}^{(2)}) = \frac{J_0 k_B T \varepsilon_a^2 B^{(2)2}(\mathbf{k}_\perp^{(s)},L)}{B^{(2)2}(\mathbf{k}_\perp^{(i)},0)} \left(\frac{z}{r}\right)^2 \times \frac{1}{L}\int_0^L dz\, B^{(2)2}(\mathbf{k}_\perp^{(s)},z) B^{(2)2}(\mathbf{k}_\perp^{(i)},z) \times \sum_{j=1,2} \frac{1}{K_{jj}q^{(sc)2}(z) + (K_{33}-K_{jj})[\mathbf{q}^{(sc)}(z)\cdot \mathbf{n}^0(z)]^2} \times \{[\mathbf{e}_j(\mathbf{q}^{(sc)},z)\cdot \mathbf{e}^{(2)}(\mathbf{k}_\perp^{(i)},z)][\mathbf{n}^0(z)\cdot \mathbf{e}^{(2)}(\mathbf{k}_\perp^{(s)},z)] + [\mathbf{e}_j(\mathbf{q}^{(sc)},z)\cdot \mathbf{e}^{(2)}(\mathbf{k}_\perp^{(s)},z)][\mathbf{n}^0(z)\cdot \mathbf{e}^{(2)}(\mathbf{k}_\perp^{(i)},z)]\}^2. \tag{6.9}$$

Equations (6.8) and (6.9) describe the intensity of the single light scattering by the CLC cell in the positive half space for the arbitrary director orientations at the boundaries. For the negative half space the scattering intensity is calculated in a similar way.

In obtaining Eq. (6.7) we use large parameters $\Omega = k_0/q_0$ and $\tilde{\Omega} = q_\perp^{(sc)}/q_0$. Therefore the application of the WKB approximation imposes restrictions on the scattering geometries. First of all the scattering angle γ between vectors $\mathbf{k}_\perp^{(i)}$ and $\mathbf{k}_\perp^{(s)}$ is not small ($\gamma \gg q_0/k_0 \sim \lambda/P$), since $q_\perp^{(sc)}P \gg 1$ in Eq. (6.7). Moreover, the angles between the z axis and the wave vectors of the incident and scattered waves for the extraordinary beam cannot be close to 90° due to the effect of the beam capture in the plane wave channel which was described in Sec. III. At last there is a restriction on the thickness of CLC, $L \ll k_0/q_0^2 \sim \pi P^2/\lambda$, it is the consequence of the second inequality (A33). From the latter inequality it follows that obtained equations are valid to the region of thickness from a rather thin CLC up to that containing many pitches.

We calculate the light scattering intensities $I(\mathbf{e}^{(1)},\mathbf{e}^{(2)})$ and $I(\mathbf{e}^{(2)},\mathbf{e}^{(2)})$ for the geometry shown in Fig. 4. The results are represented as the intensity distribution on a flat screen, normal to the z axis. We choose the following CLC parameters: $\varepsilon_a = 1.0$, $\varepsilon_\perp = 2.5$, $K_{11} = 3.0\times 10^{-6}$ dyn, $K_{22} = 2.0\times 10^{-6}$ dyn, $K_{33} = 5.0\times 10^{-6}$ dyn, the ratio of the CLC thickness to the pitch L/P is equal to $1/2$; the angle ϕ_i between the vector $\mathbf{k}_\perp^{(i)}$, and the vector director of the beam entering the CLC at $z=0$ is set to be $\phi_i = \pi/4$. Figure 5 shows the intensity of the scattered light for the angle of incidence $\pi/4$ with respect to the z axis. For both types of scattering the intensity is maximal in the region of small scattering angles, $\theta_{sc} = \angle(\mathbf{k}^{(s)},\mathbf{k}^{(i)}) \approx 0$. One can see that this region for $(o)-(e)$ scattering is wider than for the $(e)-(e)$ type. The intensity of $(e)-(e)$ scattering for $\mathbf{k}^{(s)} \approx \mathbf{k}^{(i)}$ formally tends to infinity whereas for the $(o)-(e)$ case it is finite. Here we do not consider the region $|\mathbf{k}^{(s)} - \mathbf{k}^{(i)}| \lesssim q_0$ [the white spot in the Fig. 5(b)], since our approach is not applicable in this region. Figure 6 shows the same intensities for the angle of incidence equal to $\pi/8$.

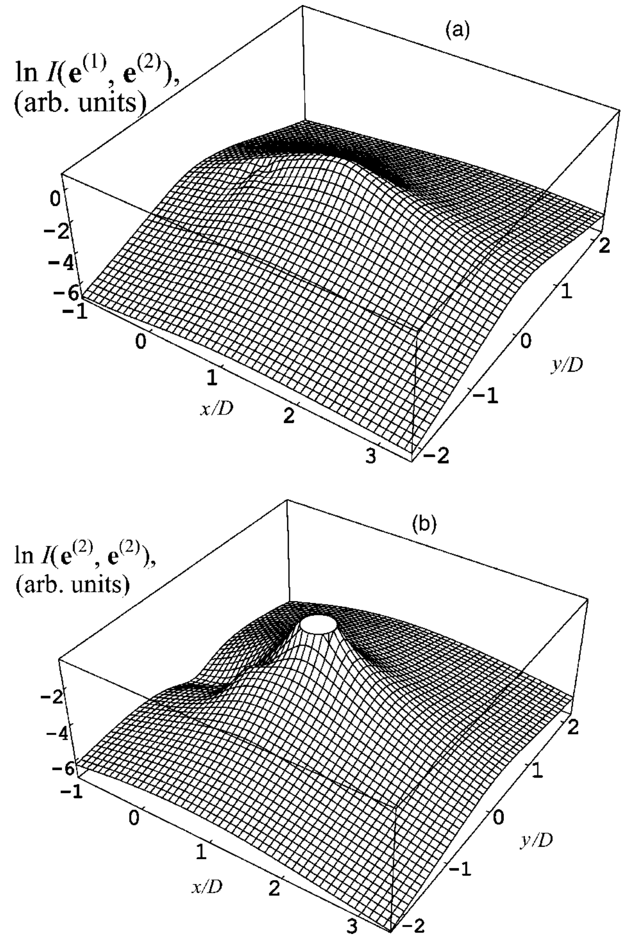
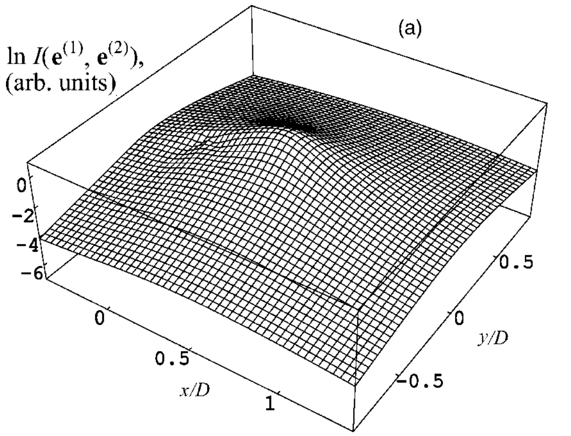


FIG. 5. The logarithm of the light scattering intensity for $(o)-(e)$, (a), and $(e)-(e)$, (b), types of scattering. All magnitudes are expressed in relative units identical for both types of scattering. Angle of incidence is equal to $\pi/4$. Coordinates x and y are measured in distances between the slab and the screen D (see Fig. 4).

Figure 7 shows the angular dependencies of the $(o)-(e)$ light scattering intensity on the slab thickness. The intensity is normalized by the scattering volume. The calculations are completed for $\mathbf{k}_\perp^{(i)} \parallel \mathbf{k}_\perp^{(s)}$ and the angle of incidence $\chi^{(i)} = \pi/3$. The pitch P is fixed and three slab thicknesses corresponding to the director twisting angles $\pi/6, \pi/3$, and $\pi/2$ are considered. One can see that dependence of the scattering indicatrix on the slab thickness is nonlinear. Note that for a linear dependence these lines should coincide.



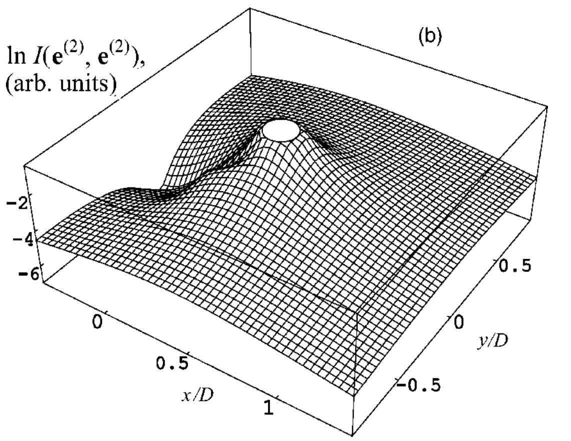


FIG. 6. The logarithm of the light scattering intensity for $(o)-(e)$, (a), and $(e)-(e)$, (b), types of scattering. The angle of incidence is equal to $\pi/8$. The other magnitudes are the same as in Fig. 5.

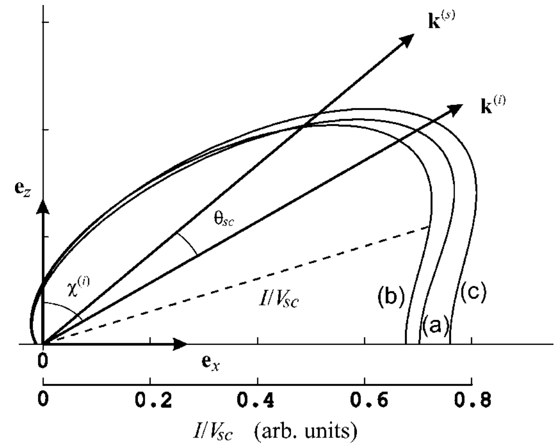


FIG. 7. Indicatrix of the $(o)-(e)$ scattering in the xz plane. The dependence of intensity on the direction of the wave vector of the scattered wave is shown. The part of the indicatrix corresponding to the region $z>0$ is presented. The curves are calculated for different slab thicknesses L: (a) $P/6$; (b) $P/3$; (c) $P/2$. Here $\varepsilon_{\parallel}, \varepsilon_{\perp}$ and Frank modules K_{ii} are the same as in Fig. 5; $\mathbf{e}_x$ and $\mathbf{e}_z$ are directions of the x and z axes. Dotted line is the intensity I expressed in arbitrary units and normalized by the scattering volume $V_{\rm sc}$.

VII. EXTINCTION

When the beam propagates in the fluctuating medium it loses the energy due to scattering and these losses determine the extinction coefficient σ. This coefficient can be calculated by integration over all angles of scattering. In the homogeneous medium the damping of the beam has an exponential form

$$\mathcal{I}(\ell)=\mathcal{I}(0)\exp(-\sigma\ell), \tag{7.1}$$

where ℓ is the path of the beam. In the inhomogeneous medium the extinction coefficient depends on the coordinates and Eq. (7.1) is substituted by

$$\mathcal{I}(\ell)=\mathcal{I}(0)\exp\left(-\int_0^{\ell}\sigma(l)dl\right), \tag{7.2}$$

where dl is an element of the beam trajectory. The local coefficient $\sigma(\ell)$ in Eq. (7.2) is determined by the total (integral) cross section of the scattering. According to Eq. (6.7) the scattering intensity from a narrow layer dz coincides with the similar relation for nematic liquid crystals (NLC), so the coefficient σ can be calculated using the known results for NLC [36–38].

In a homogeneous anisotropic media there are two extinction coefficients $\sigma^{(1)}$ and $\sigma^{(2)}$. In the Born approximation they are equal to

$$\sigma^{(i)}=\frac{k_0^4}{16\pi^2}\frac{e_\alpha^{(i)}e_\beta^{(i)}}{n_{(i)}\cos\,\delta_{(i)}} \times\sum_{s=1,2}\int d\Omega_{\mathbf{k}^{(s)}}\frac{n_{(s)}e_\mu^{(s)}e_\nu^{(s)}}{\cos^2\delta_{(s)}}\mathcal{G}_{\alpha\mu\beta\nu}(\mathbf{k}^{(i)}-\mathbf{k}^{(s)}) \tag{7.3}$$

with $i=1$ for ordinary and $i=2$ for extraordinary beams, $\int d\Omega_{\mathbf{k}^{(s)}}$ is the integration over all directions of the unit vector $\mathbf{k}^{(s)}/k^{(s)}$. In our system the values $\mathbf{e}^{(j)}, \delta_{(j)}, n_{(j)}$, and $\hat{\mathcal{G}}$ in the right hand side of Eq. (7.3) depend on z, therefore $\sigma^{(i)}=\sigma^{(i)}(\mathbf{k}_{\perp}^{(i)},z)$.

As far as our expression for the correlation function Eq. (A32) is valid for $q\gg q_0$ [Eq. (A23)] the domain of integration in Eq. (7.3) have to be restricted by $q=|\mathbf{k}^{(s)}-\mathbf{k}^{(i)}|\gtrsim q_0$. For the ordinary beam $(i=1)$ this cutoff is not essential since the term with $s=1$ is equal to zero and for the term with $s=2$ the scattering vector $\mathbf{k}^{(s)}-\mathbf{k}^{(i)}\neq 0$ in general. For the extraordinary beam $(i=2)$ the integral with $s=2$ for the correlation function (A32) diverges logarithmically for small scattering angles. For NLC this fact is well known [37,38]. Therefore the cutoff is essential in this term. Note that the region $q\ll q_0$ contrary to the case of NLC does not noticeably contribute to $\sigma^{(2)}$. The reason is that the correlation function of the director fluctuations in CLC for $q\ll q_0$ has smecticlike behavior [17] and the integral (7.3) for $|\mathbf{k}^{(s)}-\mathbf{k}^{(i)}|\to 0$ converges. Therefore the region $|\mathbf{k}^{(s)}-\mathbf{k}^{(i)}|\gtrsim q_0$ makes the main contribution to the extinction and Eq. (7.3) with such a cutoff is reasonable approximation for the extinction coefficient in CLC with the large pitch.

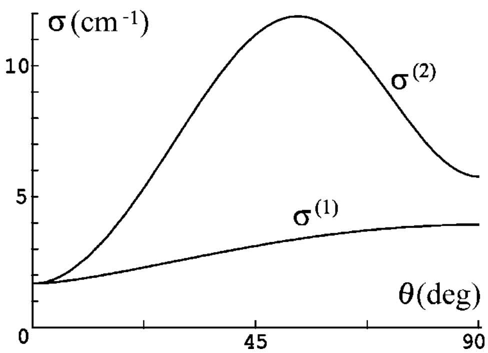


FIG. 8. The extinction coefficients $\sigma^{(1,2)}(\theta)$ for $\lambda=0.6328\ \mu$m in CLC with the same $\varepsilon_{\parallel}, \varepsilon_{\perp}$ and K_{ii} as in Fig. 5.

The element of the trajectory length dl in Eq. (7.2) is a function of $\mathbf{k}_{\perp}^{(i)}$ and z. For ordinary beam it depends on $k_{\perp}$ only,

$$dl = dzF^{(1)}(k_{\perp}),$$

where

$$F^{(1)}(k_{\perp}) = k^{(1)}/k_z^{(1)}(k_{\perp}) = (1 - \varepsilon_{\perp}^{-1}k_0^{-2}k_{\perp}^2)^{-1/2}.$$

For the extraordinary beam

$$dl = dzF^{(2)}(\mathbf{k}_{\perp},z),$$

where

$$F^{(2)}(\mathbf{k}_{\perp},z) = \sqrt{1 + \left|\frac{d\mathbf{r}_{\perp}(z)}{dz}\right|^2}.$$

The ratio $d\mathbf{r}_{\perp}/dz$ is determined by Eq. (3.16), so we get

$$\mathcal{I}^{(j)}(\ell) = \mathcal{I}^{(i)}(0)\exp\left(-\int_0^{\ell} dz\sigma^{(j)}(\mathbf{k}_{\perp},z)F^{(j)}(\mathbf{k}_{\perp},z)\right). \tag{7.4}$$

The functions $\sigma^{(i)}(\mathbf{k}_{\perp}^{(i)},z)$ can be rewritten as functions of a single scalar variable $\theta(z)$, the angle between $\mathbf{k}^{(i)}(z)$ and $\mathbf{n}^0(z)$, i.e., $\sigma^{(i)}=\sigma^{(i)}(\theta)$. The cause is that $\sigma^{(i)}(\mathbf{k}_{\perp}^{(i)},z)$ in Eq. (7.3) is a local value and the local symmetry of CLC coincides with the NLC symmetry. Figure 8 shows the dependence of the extinction coefficients $\sigma^{(1,2)}$ on θ.

VIII. CONCLUSION

We have considered the problem of light propagation and scattering in cholesterics with the large pitch. From the optical point of view this system presents a stratified medium and solution of the scattering problem differs significantly from the case of the homogeneous system. In the homogeneous system the incident beam is a plane wave, the scattered wave is considered in the Fraunhofer zone and as a result the light scattering intensity is determined by the three-dimensional Fourier image of the permittivity fluctuations.

In a helical medium the incident plane wave is transformed inside the scattering system into a normal wave of the helical medium which is not plane. The Green's function, i.e., the field of a point source, has a complex form as well. In particular, it depends on positions of the source and the receiver separately. Moreover, the Green's function has forbidden zones, i.e., regions where the wave cannot penetrate. These features are caused by the variation of the optical parameters in the medium. As a result the trajectory and the polarization vector of the wave change in a rather complicated way. Besides, for certain directions the wave returns back and as a result a wave guide propagation takes place. This causes, in particular, an unusual behavior of the light scattering intensity as far as the scattered waves can be captured by the wave channel. Finally, the spatial correlation function of the permittivity tensor fluctuations resulting from the director fluctuations, as well as the Green's function, is not determined by the difference of the coordinates only, but essentially depends on their projections to the helical axis.

In the present paper we have analyzed all these factors for CLC with the large pitch and have obtained the expressions for the light scattering intensity in a closed form. They differ essentially from the expressions for the spatially homogeneous media. First of all they represent integrals over all layers of CLC medium along the helical axis. The light scattering intensity, similar to the case of nematic liquid crystal, depends on the Frank modules, permittivity tensor, director orientations on the boundaries, and the wave vector directions of the incident and the scattered waves. An additional parameter is the cholesteric pitch. As a result of the presence of the large-scale periodic structure the dependence of the light scattering intensity on the volume is nonmonotonic and the damping due to scattering is not described by the Buger law.

Using the typical parameters of CLC we have calculated the angular dependance of the light scattering intensity. The obtained dependencies are less sharp in comparison to NLC, where the correlation length is infinite [27]. The reason is the fluctuation damping due to emergence of the characteristic size P in the system which plays a role of the finite correlation length. Nevertheless, the light scattering intensity changes significantly with the scattering angle and depends on the orientation of the wave vectors of the incident and the scattered waves and on the direction of the helical axis.

Experimental investigations of light scattering in CLC with the large pitch make it possible to clarify the whole set of interesting problems. First of all, there exists a problem of behavior of the $(e)-(e)$-type scattering intensity for the small angles θ_{sc}. In nematics this value tends to infinity as $\theta_{\mathrm{sc}} \to 0$. In cholesterics the correlation function varies from the nematiclike to the smecticlike for small scattering angles [17]. As a result the light scattering intensity becomes less singular for $\theta_{sc}\to 0$. Thus measuring the angular dependence of the light scattering intensity it is possible to investigate the transition from one regime to another one. Second, studying the light scattering in rather thick CLC samples provides a possibility of observing penetration of the scattered light into the wave channel. And, finally, investigating the vicinity of the turning point permits us to determine the specific features of light scattering near caustic.

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APPENDIX A: CORRELATION FUNCTION OF THE DIRECTOR FLUCTUATIONS IN CLC WITH THE LARGE PITCH

In order to calculate the correlation function of the director fluctuations in the Gaussian approximation we restrict ourself to the quadratic terms over $\delta\mathbf{n}$ in the free energy (2.1),

$$\delta F = \frac{1}{2}\int d\mathbf{r}\{K_{11}(\boldsymbol{\nabla}\cdot\delta\mathbf{n})^2 + K_{22}[\mathbf{n}^0\cdot(\boldsymbol{\nabla}\times\delta\mathbf{n})]^2 + K_{33}[(\delta\mathbf{n}\cdot\boldsymbol{\nabla})\mathbf{n}^0 + (\mathbf{n}^0\cdot\boldsymbol{\nabla})\delta\mathbf{n}]^2\}. \tag{A1}$$

Here we take into account the relations $\operatorname{div}\mathbf{n}^0=0$ and $\operatorname{curl}\mathbf{n}^0=-q_0\mathbf{n}^0$ which are valid for the helical structure (2.2). The vector $\delta\mathbf{n}=(\delta n_x,\delta n_y,\delta n_z)$ can be parametrized by using two functions. As far as $|\mathbf{n}|=|\mathbf{n}^0|=1$ the condition $\delta\mathbf{n}\perp\mathbf{n}^0$ is valid in the first order in δn. Therefore for CLC this parametrization has the form [17,18]

$$\delta n_x(\mathbf{r}) = -u_1(\mathbf{r})\sin\phi(z),$$

$$\delta n_y(\mathbf{r}) = u_1(\mathbf{r})\cos\phi(z),$$

$$\delta n_z(\mathbf{r}) = u_2(\mathbf{r}). \tag{A2}$$

The modes u_1 and u_2 determine the director fluctuations in the xy plane and along the z axis, respectively (Fig. 9). In vector notations we can write

$$\delta\mathbf{n}(\mathbf{r}) = u_1(\mathbf{r})\mathbf{h}^{(1)}(z) + u_2(\mathbf{r})\mathbf{h}^{(2)}, \tag{A3}$$

where

$$\mathbf{h}^{(1)}(z) = \mathbf{h}^{(2)}\times\mathbf{n}^0(z), \quad \mathbf{h}^{(2)} = \mathbf{e}_z, \tag{A4}$$

and $\mathbf{e}_z$ is the unit vector directed along the z axis.

From Eq. (A3) we can express the correlation function of the director fluctuations through the correlation matrix of the scalar functions $u_{1,2}$,

$$g_{\alpha\beta}(\mathbf{r}_\perp;z_1,z_2) = \sum_{k,l=1}^{2} G_{kl}(\mathbf{r}_\perp;z_1,z_2)h_\alpha^{(k)}(z_1)h_\beta^{(l)}(z_2), \tag{A5}$$

where

$$G_{kl}(\mathbf{r}_{1\perp}-\mathbf{r}_{2\perp};z_1,z_2) \equiv G_{kl}(\mathbf{r}_1,\mathbf{r}_2) = \langle u_k(\mathrm{r}_1)u_l(\mathbf{r}_2)\rangle. \tag{A6}$$

As far as in equilibrium CLC is spatially homogeneous in the plane normal to the z axis we use a two-dimensional Fourier transformation. Substituting Eq. (A2) into Eq. (A1) and completing two-dimensional Fourier transformation we can get the distortion energy in the form

$$\delta F = \int \frac{d^2q}{(2\pi)^2}\delta F_{\mathbf{q}}, \tag{A7}$$

where

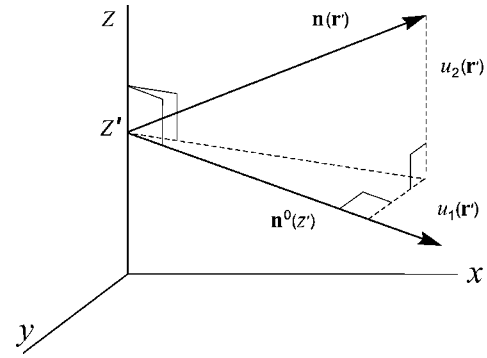

FIG. 9. Fluctuation modes $u_{1,2}$ in CLC.

$$\delta F_{\mathbf{q}} = \frac{1}{2}\int dz\{K_{11}|\partial_z u_2 + i(-\sin\phi\, q_x + \cos\phi\, q_y)u_1|^2 + K_{22}|-\partial_z u_1 + iu_2(\cos\phi\, q_y - \sin\phi\, q_x)|^2 + K_{33}[|u_2q_0 + i(\cos\phi\, q_x + \sin\phi\, q_y)u_1|^2 + |u_2|^2(\cos\phi\, q_x + \sin\phi\, q_y)^2]\}. \tag{A8}$$

Integrating by parts and omitting the terms outside the integral we present the value $\delta F_{\mathbf{q}}$ as a quadratic form

$$\delta F_{\mathbf{q}} = \frac{1}{2}\int \mathbf{u}^*(\mathbf{q},z)\hat{\mathcal{A}}(\mathbf{q},z)\mathbf{u}(\mathbf{q},z)dz \tag{A9}$$

with

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}.$$

The matrix $\hat{\mathcal{A}}$ is a differential operator of the second order. In the coordinate frame with the x axis directed along the $\mathbf{q}$ vector ($q_x=q$, $q_y=0$) it has the form

$$\hat{\mathcal{A}} = K_{11}\begin{pmatrix} q^2\sin^2\phi & iq\sin\phi\partial_z \\ iq\partial_z\sin\phi & -\partial_z^2 \end{pmatrix} + K_{22}\begin{pmatrix} -\partial_z^2 & -iq\partial_z\sin\phi \\ -iq\sin\phi\partial_z & q^2\sin^2\phi \end{pmatrix} + K_{33}\begin{pmatrix} q^2\cos^2\phi & -iq_0q\cos\phi \\ iq_0q\cos\phi & q^2\cos^2\phi + q_0^2 \end{pmatrix}, \tag{A10}$$

where $\partial_z^2 \equiv \partial^2/\partial z^2$.

The probability of fluctuations is proportional to $\exp[-\delta F_{\mathbf{q}}/k_BT]$ where k_B is the Boltzmann constant and T is temperature. As it follows from general principles of statistical mechanics [39] the calculation of the correlation function leads to inversion of the $\hat{\mathcal{A}}$ matrix. This procedure is equivalent to solution of the equation

$$\hat{\mathcal{A}}(\mathbf{q},z)\hat{G}(\mathbf{q};z,z_1) = k_BT\delta(z-z_1)\hat{I}. \tag{A11}$$

For unambiguous solution Eq. (A11) has to be complemented by boundary conditions. In the infinite system the principle of the correlations decay, $\hat{G}(\mathbf{q};z,z_1)\to\hat{0}$ for $z\to\pm\infty$, should be used as such conditions.

The correlation function of the director fluctuations $\hat{G}(z,z_1)$ in CLC with the large pitch was considered in detail in Ref. [26]. The matrix $\hat{G}$ obeys the inhomogeneous system (A11) of two differential equations with periodic coefficients and decay condition at $z \to \pm\infty$. Note that for $z \neq z_1$ Eq. (A11) is homogeneous. Primarily we solve the homogeneous equations for $z > z_1$ and $z < z_1$ and then we construct the correlation function using the continuity condition for $\hat{G}$ and the jump of its derivative for $z = z_1$.

The system of homogeneous equations has the form

$$\Bigg[-\begin{pmatrix} K_{22} & 0 \\ 0 & K_{11} \end{pmatrix} \frac{d^2}{d\xi^2} + i\tilde{\Omega}(K_{11}-K_{22})\sin\phi \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{d}{d\xi} + \begin{pmatrix} \tilde{\Omega}^2(K_{11}\sin^2\phi + K_{33}\cos^2\phi) & -i\tilde{\Omega}\cos\phi(K_{22}+K_{33}) \\ i\tilde{\Omega}\cos\phi(K_{11}+K_{33}) & \tilde{\Omega}^2(K_{22}\sin^2\phi + K_{33}\cos^2\phi) + K_{33} \end{pmatrix} \Bigg] \mathbf{u}(\xi) = 0, \quad \text{(A12)}$$

where $\tilde{\Omega} = q/q_0$, $\xi = q_0 z$. The system of two differential equations of the second order Eq. (A12) has four linearly independent solutions. Let us construct two matrices $\hat{u}_1(\xi)$ and $\hat{u}_2(\xi)$ with the properties $\hat{u}_1(\xi) \to \hat{0}$ at $\xi \to +\infty$, $\hat{u}_2(\xi) \to \hat{0}$ at $\xi \to -\infty$ using four linearly independent vector-column solutions of Eq. (A12). Such a selection provides the required behavior of $\hat{G}(\xi,\xi_1)$ at infinity.

The correlation function is sought in the form

$$\hat{G}(\xi,\xi_1) = \begin{cases} \hat{u}_1(\xi)\hat{v}_1(\xi_1) & \text{for } \xi \geqslant \xi_1, \\ \hat{u}_2(\xi)\hat{v}_2(\xi_1) & \text{for } \xi < \xi_1, \end{cases} \quad \text{(A13)}$$

where $\hat{v}_1$ and $\hat{v}_2$ are 2×2 matrices. We have

$$\hat{G}(\xi,\xi_1) = k_B T q_0^{-1} \begin{cases} \hat{u}_1(\xi)\hat{u}_1^{-1}(\xi_1)\hat{w}(\xi_1)\hat{K}^{-1} & \text{for } \xi \geqslant \xi_1, \\ \hat{u}_2(\xi)\hat{u}_2^{-1}(\xi_1)\hat{w}(\xi_1)\hat{K}^{-1} & \text{for } \xi < \xi_1, \end{cases} \quad \text{(A14)}$$

where $\hat{w}(\xi) = [\hat{u}_2'(\xi)\hat{u}_2^{-1}(\xi) - \hat{u}_1'(\xi)\hat{u}_1^{-1}(\xi)]^{-1}$,

$$\hat{K} = \begin{pmatrix} K_{22} & 0 \\ 0 & K_{11} \end{pmatrix}.$$

We consider the CLC with large pitch of helix, that implies the case $q \gg q_0$, and get the solutions $\mathbf{u}$ of the homogeneous equation (A12) using a large parameter $\tilde{\Omega} \gg 1$. Using the vector WKB method we get

$$\hat{u}_1(\xi) = \begin{pmatrix} -i\mu_1^{-1}\sin\phi & 1 \\ -1 & -i\mu_2^{-1}\sin\phi \end{pmatrix} \hat{\Phi}_-,$$

$$\hat{u}_2(\xi) = \begin{pmatrix} i\mu_1^{-1}\sin\phi & -1 \\ -1 & -i\mu_2^{-1}\sin\phi \end{pmatrix} \hat{\Phi}_+, \quad \text{(A15)}$$

where phase factors $\hat{\Phi}_\pm = \hat{\Phi}_\pm(\xi,\xi_0)$,

$$\hat{\Phi}_\pm(\xi,\xi_0) = \frac{|\cos(\xi_0+\phi_0)|}{|\cos\phi|} \widehat{\mathrm{diag}} \left\{ \sqrt{\frac{\mu_l(\xi)}{\mu_l(\xi_0)}} \times \exp\left(\pm\tilde{\Omega} \int_{\xi_0}^{\xi} \mu_l(\xi') d\xi' \right) \right\}, \quad \text{(A16)}$$

$\widehat{\mathrm{diag}}(x_l)$ denotes the diagonal matrix with elements x_1, x_2 on its main diagonal, $\phi = \phi(\xi) = \xi + \phi_0$, $\mu_l = \mu_l(\xi)$,

$$\mu_l(\xi) = \sqrt{\sin^2\phi(\xi) + \frac{K_{33}}{K_{ll}}\cos^2\phi(\xi)}, \quad l = 1,2. \quad \text{(A17)}$$

If we get back from dimensionless variable ξ to the variable z we finally have for the $\hat{G}$ matrix

$$\hat{G}(\mathbf{q};z_1,z_2) = \hat{G}_1(\mathbf{q};z_1,z_2) + \hat{G}_2(\mathbf{q};z_1,z_2), \quad \text{(A18)}$$

where

$$(G_j)_{kl}(\mathbf{q};z_1,z_2) = \frac{k_B T}{2qK_{33}\cos\phi(z_1)\cos\phi(z_2)} \times \exp\left(-q \left| \int_{z_1}^{z_2} \mu_j(z) dz \right| \right) \times \ell_k^{(j)}(\mathbf{q};z_1,z_2)\ell_l^{(j)*}(\mathbf{q};z_2,z_1), \quad \text{(A19)}$$

where $\cos\phi = \mathbf{q}\cdot\mathbf{n}^0/q$, $\sin\phi = \sqrt{q^2 - (\mathbf{q}\cdot\mathbf{n}^0)^2}/q$,

$$\ell^{(1)}(\mathbf{q};z,z') = \left(i\,\mathrm{sgn}(z-z')\frac{\sin\phi(z)}{\sqrt{\mu_1(z)}}, \sqrt{\mu_1(z)} \right),$$

$$\ell^{(2)}(\mathbf{q};z,z') = \left(\sqrt{\mu_2(z)}, i\,\mathrm{sgn}(z'-z)\frac{\sin\phi(z)}{\sqrt{\mu_2(z)}} \right). \quad \text{(A20)}$$

Summing over k and l in Eq. (A5) and using Eqs. (A4) and (A18)–(A20) we get the correlation function of the director fluctuations in the form

$$g_{\alpha\beta}(\mathbf{q};z_1,z_2) = \frac{k_BT}{2qK_{33}\cos\phi(z_1)\cos\phi(z_2)} \times \sum_{j=1}^{2} \exp\left(-q\left|\int_{z_1}^{z_2}\mu_j(z)dz\right|\right) \times f_\alpha^{(j)}(\mathbf{q};z_1,z_1-z_2)f_\beta^{(j)*}(\mathbf{q};z_2,z_2-z_1), \tag{A21}$$

where $\mathbf{f}^{(j)}(\mathbf{q};z,z-z')=\Sigma_{k=1,2}\ell_k^{(j)}(\mathbf{q};z,z')\mathbf{h}^{(k)}(z)$. In the coordinate system used in Eq. (A10) vectors $\mathbf{f}^{(j)}$ have the form

$$\mathbf{f}^{(1)}(\mathbf{q};z,z_-) = \frac{\text{sgn}(z_-)}{\sqrt{\mu_1(z)}} \times \left(i\sin^2\phi(z), -\frac{i}{2}\sin 2\phi(z), \text{sgn}(z_-)\mu_1(z)\right),$$

$$\mathbf{f}^{(2)}(\mathbf{q};z,z_-) = \sqrt{\mu_2(z)} \times \left(-\sin\phi(z), \cos\phi(z), \text{sgn}(z_-)\frac{i\sin\phi(z)}{\mu_2(z)}\right). \tag{A22}$$

Note that the correlation function (A21) grows in points $z_{1,2}$ where $\cos\phi(z_{1,2})$ in the denominator of Eq. (A19) tends to zero. These points occur in the region where the WKB approximation is violated and Eq. (A21) is inapplicable.

The range of applicability of the WKB approximation is determined by three inequalities. The first inequality,

$$\tilde{\Omega}\mu_l \gg 1, \tag{A23}$$

implies that phase factors in Eq. (A16) are rapidly varying values. The second inequality impose limitations on the proximity of the eigenvalues μ_1 and μ_2 in the whole interval from ξ_0 up to ξ,

$$\min_{\xi_0 \le \xi' \le \xi} |\mu_1(\xi') - \mu_2(\xi')| \gg \tilde{\Omega}^{-1}. \tag{A24}$$

Finally the third inequality implies smallness of the next correction to the exponential terms in Eq. (A16) for any ξ_0,ξ and imposes restrictions on the width of the region $\xi-\xi_0$ where the WKB formula is applicable,

$$|\xi - \xi'| \ll \min(\overline{\mu_1},\overline{\mu_2})\tilde{\Omega}, \tag{A25}$$

where $\overline{\mu_l}$ is the average of μ_l within $[\xi_0;\xi]$ interval.

Let us discuss the applicability range of Eq. (A21). The first condition, Eq. (A23), holds true in our case since the value $\tilde{\Omega}$ is a large parameter and by virtue of Eq. (A17) $\mu_{1,2}\sim 1$ for $K_{33}/K_{ll}\sim 1$. Therefore there remain two possible restrictions due to inequalities (A24) and (A25). As far as $\mu_{1,2}\sim 1$ it follows from inequality (A25) that $|z_1-z_2| \ll q/q_0^2 \sim \tilde{\Omega}P$.

The limitation (A24) is the most essential since the eigenvalues μ_1 and μ_2 coincide for $\cos\phi=0$. Therefore Eq. (A21) loses its sense if the point z_* with $\cos\phi(z_*)=0$ is situated between points z_1 and z_2. As far as $\mu_1=\mu_2$ identically for $K_{11}=K_{22}$ it is essential that inequality $|K_{11}-K_{22}| \gg 2(K_{11}K_{22}/K_{33})(q_0/q)$ has to be fulfilled. In particular it is illegal in Eqs. (A21) to use the equal constant approximation for the Frank energy (2.1).

Let us estimate the region where inequality (A24) is valid. For this purpose we introduce a new variable $\zeta=q_0z+\phi_0-\pi/2=\phi-\pi/2$ and expand μ_l near the point $\zeta=0$,

$$\mu_l \approx 1 + C_l\zeta^2, \tag{A26}$$

where $C_l=(K_{33}-K_{ll})/2K_{ll}$, $l=1,2$. In this case the condition (A24) gives

$$\tilde{\Omega}|\zeta|^3 \gg 1. \tag{A27}$$

It means that the expression (A21) is valid if $\tilde{\Omega}|\zeta|^3\gg 1$, $\tilde{\Omega}|\zeta_1|^3\gg 1$ and there are no points with coinciding μ_1 and μ_2 between ζ and ζ_1 where $\cos\phi=0$.

Analysis of the correlation function behavior in the vicinity of points with $\cos\phi=0$ is based on methods which are used for investigation of the turning points in the WKB method. This problem we have discussed in detail in Ref. [26]. It is shown there that the correlation function is finite in the points with $\cos\phi(z_1)=0$ and $\cos\phi(z_2)=0$.

Now we are going to construct the function $\hat{\mathcal{G}}(\mathbf{q}^{(sc)}(z_+),z_+)$, which enters the expression for the light scattering intensity. We shall perform a procedure analogous to that as in obtaining of Eq. (6.6). In all slowly varying factors of $\hat{\mathcal{G}}(\mathbf{q};z_1,z_2)$ we substitute z_1 and z_2 by z_+. We restrict ourselves by the term linear in z_- in the exponential factors and also keep factors containing the function sgn(z_-). In this case Eq. (2.13) takes the form

$$\mathcal{G}_{\alpha\beta\gamma\delta}(\mathbf{q}^{(sc)}(z),z) = \varepsilon_a^2[n_\alpha^0(z)n_\gamma^0(z)g_{\beta\delta}(\mathbf{q}^{(sc)}(z),z) + n_\alpha^0(z)n_\delta^0(z)g_{\beta\gamma}(\mathbf{q}^{(sc)}(z),z) + n_\beta^0(z)n_\gamma^0(z)g_{\alpha\delta}(\mathbf{q}^{(sc)}(z),z) + n_\beta^0(z)n_\delta^0(z)g_{\alpha\gamma}(\mathbf{q}^{(sc)}(z),z)], \tag{A28}$$

where

$$g_{\alpha\beta}(\mathbf{q}^{(sc)}(z),z) = \frac{k_BT}{2q_\perp^{(sc)}K_{33}\cos^2\phi(z)} \times \sum_{j=1}^{2}\int_{-\infty}^{\infty} dz_-\exp[iq_z^{(sc)}(z)z_- - q_\perp^{(sc)}\mu_j(z)|z_-|] \times f_\alpha^{(j)}(\mathbf{q}_\perp^{(sc)};z,z_-)f_\beta^{(j)}(\mathbf{q}_\perp^{(sc)};z,z_-). \tag{A29}$$

In Eqs. (A29) and (A22) we take into account the order of arguments and complex conjugation of the $\mathbf{f}^{(j)}$ functions in Eq. (A21).

Performing integration over z_- and summation over j in Eq. (A29) we get

$$g_{\alpha\beta}(\mathbf{q}^{(sc)}(z),z) = k_B T \sum_{j=1}^{2} \frac{e_{j\alpha}(\mathbf{q}^{(sc)},z)e_{j\beta}(\mathbf{q}^{(sc)},z)}{K_{jj}[q^{(sc)2} - (\mathbf{q}^{(sc)}\cdot\mathbf{n}^0)^2] + K_{33}(\mathbf{q}^{(sc)}\cdot\mathbf{n}^0)^2}, \tag{A30}$$

where

$$\mathbf{e}_2(\mathbf{q}^{(sc)},z) = \frac{\mathbf{q}^{(sc)}\times\mathbf{n}^0}{\sqrt{q^{(sc)2} - (\mathbf{q}^{(sc)}\cdot\mathbf{n}^0)^2}},$$

$$\mathbf{e}_1(\mathbf{q}^{(sc)},z) = \mathbf{n}^0 \times \mathbf{e}_2(\mathbf{q}^{(sc)},z). \tag{A31}$$

Here $\mathbf{q}^{(sc)}=\mathbf{q}^{(sc)}(z)$, $\mathbf{n}^0=\mathbf{n}^0(z)$.

Substituting Eq. (A30) into Eq. (A28) we get the correlation function of permittivity fluctuation in CLC,

$$\mathcal{G}_{\rho\nu\varphi\mu}(\mathbf{q}^{(sc)}(z),z) = \sum_{j=1}^{2} \frac{k_B T \varepsilon_a^2}{K_{jj}q^{(sc)2} + (K_{33}-K_{jj})[\mathbf{q}^{(sc)}\cdot\mathbf{n}^0(z)]^2} \times [e_{j\nu}(\mathbf{q}^{(sc)},z)\mathbf{n}^0_\rho(z) + e_{j\rho}(\mathbf{q}^{(sc)},z)\mathbf{n}^0_\nu(z)] \times [e_{j\mu}(\mathbf{q}^{(sc)},z)\mathbf{n}^0_\varphi(z) + e_{j\varphi}(\mathbf{q}^{(sc)},z)\mathbf{n}^0_\mu(z)]. \tag{A32}$$

Note, that the correlation function $\hat{g}(\mathbf{q}^{(sc)},z)$, Eq. (A30), for any fixed z coincides with $\hat{g}(\mathbf{q}^{(sc)})$ in NLC [27] if we formally put $\mathbf{q}^{(sc)}=\mathbf{q}^{(sc)}(z)$ and $\mathbf{n}^0=\mathbf{n}^0(z)$ in the nematic correlation function. This fact has the transparent physical meaning. The expression (A30) was obtained by the WKB method using the large parameter $\tilde{\Omega}\gg 1$, i.e., $q_\perp^{(sc)}P\gg 1$. In this case the correlation function (A21) is not negligibly small in the region $|z_1-z_2|\ll P$ only. For such scales CLC locally coincides with NLC.

Another feature of Eq. (A30) is the absence of divergence which is present in Eq. (A21) for $\cos\phi(z)=0$. This difference is related to the condition $|z_1-z_2|\ll P$ used for obtaining of Eq. (A30). If one of the points $z_{1,2}$ in Eq. (A21) belongs to the region where $\cos\phi(z)=0$ then the second point belongs to the same region too. In this case singularities in terms $j=1, 2$ of the correlation function are canceled. This is evident from the fact that in Eq. (A19) exponents $\mu_1(z),\mu_2(z)\to 1[\cos\phi(z_1),\cos\phi(z_2)\to 0$ consequently] and in the limit $z_1\to z_2$ the condition $\Sigma_{j=1,2}\ell_k^{(j)}(\mathbf{q};z_1,z_2)\ell_l^{(j)*}(\mathbf{q};z_2,z_1)\to 0$ is fulfilled.

Thus in the problem of light scattering it is possible to restrict ourself to the expression (A32) for the correlation function if the inequalities

$$q \gg q_0, \quad |z_1 - z_2| \ll q/q_0^2 \tag{A33}$$

are fulfilled.

APPENDIX B: SURFACE CORRECTIONS FOR LIGHT SCATTERING

For calculation of the scattering intensity in the Born approximation the Green's function $\hat{T}(\mathbf{r},\mathbf{r}')$ in the far zone is usually used and the Fraunhofer approximation, Eq. (5.6), is applied (see, e.g., Ref. [29]). If the observation point is located outside the scattering volume then the refraction at the boundaries should be taken into account. Such an approach is convenient for spatially homogeneous media and in this case $\hat{T}(\mathbf{r},\mathbf{r}')=\hat{T}(\mathbf{r}-\mathbf{r}')$.

For layered scattering systems there is no simple expression for the Green's function $\hat{T}(\mathbf{r},\mathbf{r}')$ in the coordinate representation. However, this function is known in the mixed $(\mathbf{q}_\perp,z)$ representation in the WKB approximation [Eq. (4.3)].[2] This is why we calculated the scattered field outside the layered medium using the Kirchhoff method.

Though these methods are conceptually different we show that for the spatially homogeneous scattering medium both approaches give coinciding results.

Let us consider at first an isotropic system. The scattering intensity is the modulus of the Poynting vector (3.5) which for isotropic medium is

$$\mathbf{S} = \frac{c}{8\pi}n|E|^2\frac{\mathbf{k}}{k}.$$

Let the incident beam in Eq. (2.8) be a plane wave, $\mathbf{E}^0(\mathbf{r})=E_0^{(i)}\mathbf{e}^{(i)}\exp(i\mathbf{k}^{(i)}\mathbf{r})$. We use the Fraunhofer approximation (5.6) for the Green's function. In the Born approximation the scattered field inside the scattering medium has the form

$$E^{(s)}(\mathbf{r}) = E_0^{(i)}\frac{k_0^2 e^{ikr}}{4\pi r}\mathbf{e}^{(s)}\left(\int_{V_{sc}} d\mathbf{r}'\,\delta\hat{\varepsilon}(\mathbf{r}')e^{i(\mathbf{k}^{(i)}-\mathbf{k}^{(s)})\cdot\mathbf{r}'}\right)\mathbf{e}^{(i)} = E_0^{(i)}\frac{k_0^2 e^{ikr}}{4\pi r}\mathbf{e}^{(s)}\delta\hat{\varepsilon}(\mathbf{Q})\mathbf{e}^{(i)}. \tag{B1}$$

Here $\mathbf{Q}=\mathbf{k}^{(s)}-\mathbf{k}^{(i)}$ is the scattering vector, $\delta\hat{\varepsilon}(\mathbf{Q})$ is the three-dimensional Fourier component of the permittivity fluctuations. Hence scattering intensity inside the specimen is equal to

$$I_{\rm in} = \frac{c}{8\pi}n_{\rm in}E_0^{(i)2}\left(\frac{k_0^2}{4\pi}\right)^2\frac{V_{\rm sc}}{r^2}e_\alpha^{(s)}e_\beta^{(s)}\mathcal{G}_{\alpha\nu\beta\mu}(\mathbf{Q})e_\nu^{(i)}e_\mu^{(i)}, \tag{B2}$$

where $\hat{\mathcal{G}}(\mathbf{Q})$ is the three-dimensional Fourier component of the permittivity correlation function.

Now we introduce the energy flux inside the specimen through the small area $d\Sigma_{\rm in}$ perpendicular to the Poynting vector $\mathbf{S}$

$$dI_{\rm in} = \mathbf{S}\cdot d\mathbf{\Sigma}_{\rm in} = I_{\rm in}d\Sigma_{\rm in} = I_{\rm in}r^2 d\Omega_{\rm in}, \tag{B3}$$

where $d\Omega_{\rm in}$ is the element of the solid angle (see Fig. 10). In order to calculate the energy flux outside the specimen it is necessary to take into account the surface corrections. First, it is necessary to take into account changes of the amplitude and propagation direction of the wave owing to the refraction at the boundary. Second, the corrections manifest themselves in the difference of the solid angles inside and outside the specimen, $d\Omega_{\rm in}$ and $d\Omega_{\rm out}$, due to difference of the refractive indices.

[2]This expression is valid in the wide range of $z-z'$, in particular in the near and in the far zone.

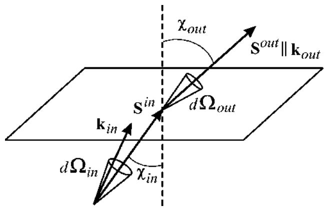

FIG. 10. Refraction on the boundary for the homogeneous scattering medium: $\mathbf{S}^{\text{in}},\mathbf{S}^{\text{out}}$ are the Poynting vectors inside and outside the scattering medium, $\mathbf{k}_{\text{in}},\mathbf{k}_{\text{out}}$ are the wave vectors, $d\Omega_{\text{in}}$ and $d\Omega_{\text{out}}$ are the elements of the solid angles, χ_{in} is the angle of incidence measured from the $\mathbf{S}^{\text{in}}$ vector, χ_{out} is the angle of refraction measured from the $\mathbf{S}^{\text{out}}$ vector. For isotropic medium $\mathbf{S}^{\text{in}}\|\mathbf{k}_{\text{in}}$ and $\mathbf{S}^{\text{out}}\|\mathbf{k}_{\text{out}}$.

Now we consider the refraction at the boundary. Consider that $E_0^{\text{out}}=t_{\text{in}\to\text{out}}E_0^{\text{in}}$, where $t_{\text{in}\to\text{out}}$ is the transfer constant of the field amplitude, which can be obtained using the Fresnel formulas. The energy flux at the boundary is equal to S_z. Inside and outside the specimen we can get

$$S_z^{\text{in}}=\frac{c}{8\pi}n_{\text{in}}E_0^{\text{in}2}\cos\chi_{\text{in}},$$

$$S_z^{\text{out}}=\frac{c}{8\pi}n_{\text{out}}E_0^{\text{in}2}t_{\text{in}\to\text{out}}^2\cos\chi_{\text{out}}. \tag{B4}$$

So the surface correction to the energy flux due to refraction has the form

$$\frac{dI_{\text{out}}}{dI_{\text{in}}}=\frac{S_z^{\text{out}}}{S_z^{\text{in}}}=\frac{n_{\text{out}}t_{\text{in}\to\text{out}}^2\cos\chi_{\text{out}}}{n_{\text{in}}\cos\chi_{\text{in}}}.$$

The variation of the solid angle can be calculated with the help of Snell's law. Let the angle of incidence of the scattered field be χ_{in} and the refraction angle be χ_{out}. The polar angles ϕ_{in} and ϕ_{out} are measured from the x axis. The element of the solid angle is equal to

$$d\Omega=\sin\chi d\chi d\phi.$$

Snell's law has the form

$$n_{\text{in}}\sin\chi_{\text{in}}=n_{\text{out}}\sin\chi_{\text{out}}. \tag{B5}$$

This law may be written for the projection of the wave vectors on the x axis,

$$n_{\text{in}}\sin\chi_{\text{in}}\cos\phi_{\text{in}}=n_{\text{out}}\sin\chi_{\text{out}}\cos\phi_{\text{out}}.$$

It follows from these equalities that $\phi_{\text{in}}=\phi_{\text{out}}$. If we differentiate Eq. (B5) we get

$$n_{\text{in}}\cos\chi_{\text{in}}d\chi_{\text{in}}=n_{\text{out}}\cos\chi_{\text{out}}d\chi_{\text{out}}. \tag{B6}$$

Taking into account the equality of the polar angles, from Eqs. (B5) and (B6) we have

$$\frac{d\Omega_{\text{in}}}{d\Omega_{\text{out}}}=\frac{n_{\text{out}}^2\cos\chi_{\text{out}}}{n_{\text{in}}^2\cos\chi_{\text{in}}}. \tag{B7}$$

Thus the cross section of the scattering intensity outside the specimen has the form

$$\begin{aligned}\frac{dI_{\text{out}}}{d\Omega_{\text{out}}}&=\frac{dI_{\text{out}}}{dI_{\text{in}}}\frac{dI_{\text{in}}}{d\Omega_{\text{in}}}\frac{d\Omega_{\text{in}}}{d\Omega_{\text{out}}}\\&=\frac{c}{8\pi}n_{\text{in}}E_0^{\text{in}2}t_{\text{in}\to\text{out}}^2\frac{k_0^4}{16\pi^2}V_{\text{sc}}\frac{n_{\text{out}}^3}{n_{\text{in}}^3}\frac{\cos^2\chi_{\text{out}}}{\cos^2\chi_{\text{in}}}\\&\quad\times e_\alpha^{(s)}e_\beta^{(s)}\mathcal{G}_{\alpha\nu\beta\mu}(\mathbf{Q})e_\nu^{(i)}e_\mu^{(i)}.\end{aligned} \tag{B8}$$

The scattering intensity calculated by the Kirchhoff method refers to the region outside the specimen. For homogeneous isotropic scattering system all values in Eq. (6.6) become independent on z, and we can put $e_\alpha^{(s)}M_{\alpha\beta}^{\text{in}\to\text{out}}e_\beta^{(s)}(\mathbf{k}_\perp^{(s)})=t_{\text{in}\to\text{out}}$, $B^{(i)2}=B^{(s)2}=k_0/k_{\text{in }z}$. Keeping in mind that $\sqrt{\varepsilon_0}=n_{\text{out}}$ we have from Eq. (6.6)

$$\begin{aligned}\frac{dI_{\text{out}}^{Kir}}{d\Omega_{\text{out}}}&=I_{\text{out}}^{Kir}r^2=\frac{c}{8\pi}n_{\text{out}}^3E_0^{\text{in}2}t_{\text{in}\to\text{out}}^2\left(\frac{k_0^2}{4\pi}\right)^2\\&\quad\times V_{\text{sc}}\left(\frac{z}{r}\right)^2\frac{k_0^2}{k_{\text{in }z}^2}e_\alpha^{(s)}e_\beta^{(s)}\mathcal{G}_{\alpha\nu\beta\mu}(\mathbf{Q})e_\nu^{(i)}e_\mu^{(i)}.\end{aligned} \tag{B9}$$

If we take into account the relations $z/r=\cos\chi_{\text{out}}$ and $k_{\text{in }z}/k_0=n_{\text{in}}\cos\chi_{\text{in}}$ then we get

$$\frac{dI_{\text{out}}}{d\Omega_{\text{out}}}=\frac{dI_{\text{out}}^{Kir}}{d\Omega_{\text{out}}}.$$

Thus for homogeneous isotropic media the both approaches are equivalent.

Now consider the case of the homogeneous anisotropic scattering medium. In this case all quantities in Eq. (6.6) do not depend on z_+ and we can compare the scattering intensity (6.6) obtained by the Kirchhoff method with that obtained for homogeneous medium in Ref. [33].

Similarly to isotropic medium it is necessary to take into account the variations of the solid angle element and the refraction at the boundary. The main problem is that the Poynting vector for the extraordinary beam is not directed along the wave vector. According to Ref. [33] the scattering intensity inside the specimen has the form

$$I_{\text{in}}=\frac{c}{8\pi}n_{(s)}E_0^{\text{in}2}\left(\frac{k_0^2}{4\pi}\right)^2\frac{V_{\text{sc}}}{r^2}\frac{f_{(s)}^2}{\cos^3\delta_{(s)}}e_\alpha^{(s)}e_\beta^{(s)}\mathcal{G}_{\alpha\nu\beta\mu}(\mathbf{Q})e_\nu^{(i)}e_\mu^{(i)}. \tag{B10}$$

Here the factor $f_{(s)}$ is determined by the Gaussian curvature

$$f_{(1)}=1,\quad f_{(2)}=\left[\frac{(\mathbf{s}^{(2)}\hat{\varepsilon}^0\mathbf{s}^{(2)})(\mathbf{s}^{(2)}\hat{\varepsilon}^{02}\mathbf{s}^{(2)})}{\varepsilon_\|\varepsilon_\perp^2}\right]^{1/2},$$

where $\mathbf{s}^{(2)}=\mathbf{S}^{(2)}/S^{(2)}$. Recalculation of the intensity in terms of external medium parameters is performed according to

$$dI_{\text{out}}=I_{\text{in}}\frac{d\Omega_{\text{in}}}{d\Omega_{\text{out}}}T_{\text{in}\to\text{out}}r^2d\Omega_{\text{out}}. \tag{B11}$$

Here the ratio $d\Omega_{\text{in}}/d\Omega_{\text{out}}$ describes the variation of the solid angles,

$$\frac{d\Omega_{\rm in}}{d\Omega_{\rm out}} = \frac{n_{\rm out}^2 \cos \chi_{\rm out}}{n_{(s)}^2 \cos \chi_{\rm in}^{(s)}} \frac{\cos^2 \delta_{(s)}}{f_{(s)}^2}. \tag{B12}$$

The factor $T_{\rm in\to out}$ determines the transfer of energy of the beam passing through the boundary,

$$T_{\rm in\to out} = \frac{n_{\rm out} \cos \chi_{\rm out}}{n_{(s)} \cos \chi_{\rm in}^{(s)}} \frac{t_{\rm in\to out}^{(s)2}}{\cos \delta_{(s)}}. \tag{B13}$$

Here $\chi_{\rm in}^{(s)}$ is the angle between the Poynting vector and the z axis.

The scattering cross section outside the specimen in the Born approximation has the form

$$\frac{dI_{\rm out}}{d\Omega_{\rm out}} = \frac{c}{8\pi} n_{(s)} E_0^{\rm in2} t_{\rm in\to out}^{(s)2} \left(\frac{k_0^2}{4\pi}\right)^2 V_{\rm sc} \frac{1}{\cos^2 \delta_{(s)}} \times \frac{n_{\rm out}^3}{n_{(s)}^3} \frac{\cos^2 \chi_{\rm out}}{\cos^2 \chi_{\rm in}^{(s)}} e_\alpha^{(s)} e_\beta^{(s)} \mathcal{G}_{\alpha\nu\beta\mu}(\mathbf{Q}) e_\nu^{(i)} e_\mu^{(i)}. \tag{B14}$$

For the case of the homogeneous medium the integral in Eq. (6.6) disappears and we get the scattering intensity obtained by the Kirchhoff method,

$$\frac{dI_{\rm out}^{Kir}}{d\Omega_{\rm out}} = \frac{c}{8\pi} n_{\rm out}^3 E_0^{\rm in2} t_{\rm in\to out}^{(s)2} \left(\frac{k_0^2}{4\pi}\right)^2 V_{\rm sc} \cos^2 \chi_{\rm out} \times B^{(s)4}(\mathbf{k}_\perp^{(s)}) e_\alpha^{(s)} e_\beta^{(s)} \mathcal{G}_{\alpha\nu\beta\mu}(\mathbf{Q}) e_\nu^{(i)} e_\mu^{(i)}. \tag{B15}$$

Comparing Eqs. (B14) and (B15) one can see that it is necessary to prove the equality

$$[B^{(s)}(\mathbf{k}_\perp^{(s)})]^2 = (\cos \delta_{(s)} n_{(s)} \cos \chi_{\rm in}^{(s)})^{-1}. \tag{B16}$$

For the ordinary beam this relation can be easily obtained using Eq. (3.13) for $B^{(s)}(\mathbf{k}_\perp^{(s)})$. As far as the directions of the wave vector and the beam vector coincide for the ordinary beam we have $[B^{(1)}(\mathbf{k}_\perp^{(s)})] = k_0/k_z^{(1)} = 1/n_{(1)} \cos \chi_{\rm in}^{(1)}$, therefore Eq. (B13) is fulfilled. For the extraordinary beam it is easy to get the following expression for $\cos \chi_{\rm in}^{(2)}$:

$$\cos \chi_{\rm in}^{(2)} = k_z^{(2)} n_{(2)} \cos \delta_{(2)} / \varepsilon_\parallel k_0.$$

Using this relation we can verify validity of Eq. (B16). Thus we prove that for the homogeneous uniaxial scattering media the Kirchhoff method is equivalent to the standard approach,

$$\frac{dI_{\rm out}}{d\Omega_{\rm out}} = \frac{dI_{\rm out}^{Kir}}{d\Omega_{\rm out}}.$$

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